6. Taking the equation in the form

$$\mathrm{sn}\,(u_1 - u_0) = \mathrm{sn}\,u_1\,\mathrm{sn}\,u_0\,\{E\,(u_1 - u_0) - c'\,(u_1 - u_0)\},$$

we at once see that

if $\qquad u_0 = 0, \quad$ then $u_1 = 2K,$

and if $\qquad u_0 = 2K, \quad$ then $u_1 = 4K.$

On account of the symmetrical undulation of the geodesic, it is sufficient to take into account values of u_0, such that

$$0 < u_0 < 2K,$$

in addition to the two limiting values already mentioned.

If we consider the curves,

$$y_1 = \frac{\mathrm{sn}\,(x - u_0)}{\mathrm{sn}\,x\;\mathrm{sn}\,u_0}, \quad y_2 = E\,(x - u_0) - c'\,(x - u_0),$$

the first value of x increasing from u_0, for which $y_1 = y_2$, is u_1

Fig. 1

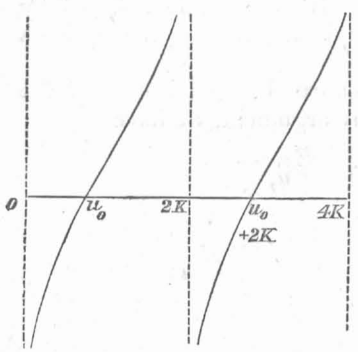
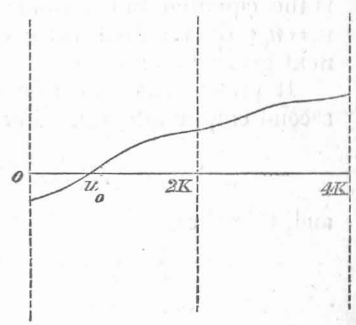


Fig. 2



Now these curves, when traced, are as in figures 1 and 2 respectively; hence

$$u_0 + 2K < u_1 < 4K.$$

If we take $\qquad u_1 - u_0 = 2K + \theta_1,$

so that $\qquad 0 < \theta_1 < 2K - u_0,$

we have

$$y_1 = \frac{\mathrm{sn}\,(2K + \theta_1)}{\mathrm{sn}\,(2K + u_0 + \theta_1)\;\mathrm{sn}\,u_0}$$

$$= \frac{\mathrm{sn}\,\theta_1}{\mathrm{sn}\,(u_0 + \theta_1)\;\mathrm{sn}\,u_0},$$

$$y_2 = 2G + E(\theta_1) - c'\theta_1;$$

and θ_1 is determined, between the above limits by the equation

$$\frac{\mathrm{sn}\,\theta_1}{\mathrm{sn}\,(u_0 + \theta_1)\;\mathrm{sn}\,u_0} = 2G + E(\theta_1) - c'\theta_1.$$

7. When the ellipticity of the oblate spheroid is small, c is small. Then G is small; and as

$$E(\theta_1) - c'\theta_1 = c\int_0^{\theta_1} \mathrm{cn}^2 u\,du,$$

it follows that $\mathrm{sn}\,\theta_1$ (and therefore θ_1) is small of the same order as c. Consequently $E(\theta_1) - c'\theta_1$ is of the second order in c; also

$$G = 2cc'\,\frac{dK}{dc}$$

$$= \tfrac{1}{4}\pi c,$$

so far as regards the most important term. Hence, when second powers of the ellipticity (and therefore second powers of c) are negligible, we have

$$\theta_1 = \tfrac{1}{2}\pi c\,\mathrm{sn}^2 u_0,$$

and consequently the excess of $u_1 - u_0$ above π is

$$\tfrac{1}{4}\pi c\,(1 + 2\,\mathrm{sn}^2 u_0)$$

to the first order of small quantities.

8. In what precedes, it has been assumed that the geodesic is not a meridian. The special case in which it is a meridian is given by $\alpha = 0$ and then $k^2 = e^2$; so that the preceding formulæ apply on the understanding that the modulus of the elliptic functions is e^2 instead of

$$\frac{e^2\cos^2\alpha}{1 - e^2\sin^2\alpha}.$$

In particular, when $u_0 = 0$ the conjugate is given by $u_1 = 2\pi$ or $\theta = \pi$: that is, the conjugate of either pole is the other pole—a result to be expected.

25 January, 1896.