

TENTAMEN

CIRCA

TRIGONOMETRIAM SPHÆROIDICAM

AUCTORE

ERASMO GEORGIO FOG THUNE

DOCTORE PHILOSOPHIAE.

H A U N I A E.

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Uti trigonometria præcipue circa triangulum per data sufficien-
tia determinandum versatur, ita pro vario trianguli situ, sive
in plano, sive in superficie sphæræ, sive in superficie sphæroï-
dis, plana, sphærica et sphæroidica perhibetur. Quum vero
planum finitum infinitæ sphæræ ad congruentiam usqve adaptari
patiatur, ipsa autem sphæra sphæroidi, excentricitate evane-
scente, manifesto æqvipolleat, patet, trigonometriam planam in
sphærica et utramqve in sphæroidica comprehendi. Cujusmodi
consideratione si hæcce haud mediocriter commendatur, neqve
minoris æstimanda videtur, perpensa utilitate singulari, qvam
terræ ac maris mensori pariter ac ipsi astronomo adfert. Vide-
licet, quum sphæroidica telluris forma ultra omne dubium po-
sita sit, generaliter intelligitur, qvoties seu coelestia seu terre-
stria investigentur loca, qvæ a solutione trianguli in superficie
telluris siti pendeant, toties exactiores eorundem determinatio-
nes, nisi triangulum perparvum sit, a trigonometria sphæroidica
petendas. Sane qvidem, si ellipsoidicam telluri formam asfig-
nare liceat, usum practicum jam commemoratum prætabit sphæ-
roidica specialior, scilicet ellipsoidica trigonometria; si autem
dubia contra probabilissimam illam telluris formam moveantur,
haud spernenda videntur, qvæcunqve in trigonometria sphæroi-
dica generaliori nobis sese offerent.

Talis tantaqve quum sit dignitas trigonometriæ sphæroidicæ, qvæ a celeberrimo Oriani (cujus scriptum ad nostras regiones nondum pervenit) et præter eum, quantum ego eqvidem scio, a nemine explanata est, curam eximiam, qva celeberrimus Besel studia mea mathematica et astronomica gubernare solitus est, vel eo luculenter declaravit, qvod in istam matheseos partem animum meum advertit, valde et vehementer incitans, ut otium, beneficio publico mihi concessum, disquisitionibus hujus generis aliquatenus impenderem. Quantum autem gaudeo, qvod difficultates complures, illæ in primis, qvas conflavit gravitas rei in articulo i explicatæ, eo jam tempore, qvo benevolæ geometræ Besel consilia adire licuit, dissipabantur; tantum doleo, mihi deinde in patriam reduci, grasante bello literis infestissimo, in longe majori complementationis parte elaboranda, vel levissima ipsius defecisse monita, qvibus, qvæ forte irrepserint, menda caveri potuisent. Veruntamen, si hoc qvalecunqve opus summi illius viri discipulo haud prorsus indignum censebitur, nihil amplius est, qvod optem.

Planum meridiani loco in superficie sphæroidis dato competentis, id ipsum dicitur planum, cui, per locum datum et centrum sphæræ, sphæroideum ibidem loci osculantis, extenso axis, circa qvem vertitur sphærois, est parallelus. Latitudo loci dicitur angulus, qvo normalis, in plano meridiani ducta, versus planum axi rotationis perpendicularē inclinatur.

Jam designando

radius sphæræ commemoratæ (radius curvaturæ meridiani)	P
radius paralleli	ρ
differentiam longitudinis loci	ω
latitudinem loci	φ

habetur

distantia illius loci a loco infinite propinquæ

$$d\Sigma = \sqrt{\rho^2 d\omega^2 + P^2 d\phi^2} \quad (\text{A})$$

Ut linea, inter duo locos in superficie sphæroidis ducta, sit brevisima, esse debet

$$\int d\Sigma = \text{minimo}$$

igitur

$$\delta \int \sqrt{\rho^2 d\omega^2 + P^2 d\phi^2} = \text{zero.}$$

Sed variatio ipsius $\int V dx$, ubi V est functio quantitatuum: $x, y, \frac{dy}{dx} = p$,

(confer. Lacroix traité element. pag. 493).

erit

$$\delta \int V dx = V \delta x + \left(\frac{dV}{dp} \right) \left\{ \delta y - p \delta x \right\} + \int \left[\left(\frac{dV}{dy} \right) - d \left(\frac{dV}{dp} \right) \right] \left(\delta y - p \delta x \right) dx$$

In casu nostro, ubi

$$V = \sqrt{r^2 + p^2 p^2}$$

$$x = \omega$$

$$y = \phi$$

$$p = \frac{d\phi}{d\omega}$$

inde seqvitur

$$\delta \int d\Sigma = o = V \delta\omega + \left(\frac{dV}{dp} \right) \{ \delta\phi - p \delta\omega \} + \int \left\{ \left(\frac{dV}{d\phi} \right) - d \left(\frac{dV}{dp} \right) \right\} (\delta\phi - p \delta\omega) d\omega.$$

Proinde, quum linea a definito puncto ad definitumducenda sit, $\delta\omega$, $\delta\phi$ in finalibus hujus lineæ punctis ad nihilum recident; et cum similitus illis exstantibus unde obtinetur

$$\left(\frac{dV}{d\phi} \right) = d \left(\frac{dV}{dp} \right) \dots \text{et minimis est inveniatur integrando} \quad (B)$$

Est generaliter

$$dV = \left(\frac{dV}{d\phi} \right) d\phi + \left(\frac{dV}{d\omega} \right) d\omega + \left(\frac{dV}{dp} \right) dp$$

hinc seqvitur

$$\left(\frac{dV}{d\phi} \right) = \frac{dV}{d\phi} - \left(\frac{dV}{d\omega} \right) \frac{1}{p} - \left(\frac{dV}{dp} \right) \frac{dp}{d\phi} = \frac{d \left(\frac{dV}{dp} \right)}{d\omega} \dots \text{modus generalis}$$

sive

$$\frac{dV}{p} - \left(\frac{dV}{d\omega} \right) \frac{d\omega}{p} - \left(\frac{dV}{dp} \right) \frac{dp}{p} = d \left(\frac{dV}{dp} \right)$$

quare subliviorum de aliis pionieris sequitur in eodem sub lege:

$$\left(\frac{dV}{d\omega} \right) d\omega = dV - \left\{ p d \left(\frac{dV}{dp} \right) + dp \left(\frac{dV}{dp} \right) \right\} = d \left\{ V - p \left(\frac{dV}{dp} \right) \right\} \dots \text{sed} \quad (C)$$

et

$$\int \left(\frac{dV}{d\omega} \right) d\omega = V - p \left(\frac{dV}{dp} \right) + \text{const.} \dots \text{et} \quad (D)$$

Hinc cursus lineæ in superficie sphæroidis innotescet. Insigniantur, nimis anguli, qvibus meridianos versus flectitur, nota α , conseqvemur:

$$V = \frac{p}{\sin. \alpha}$$

$$\text{et } d\Sigma = \frac{pd\omega}{\sin. \alpha} = \frac{Pd\phi}{\cos. \alpha}$$

$$p = \frac{P}{\rho} \cot. \alpha.$$

Quum vero

zell. Ausdehnung $\left(\frac{dV}{dp}\right) = \frac{p}{V} \cdot P^2$ nimmt während des Prozesses ab und muß also unter einer stetigen Abnahme

$$p \left(\frac{dV}{dp} \right) = p^2 \frac{\frac{p^2}{V}}{V} = \frac{V^2 - p^2}{V} = V - \frac{p^2}{V} = V - \xi \sin \alpha$$

ex (C) seqvitur

Quum autem æquatione sphæroidis data, $\left(\frac{dV}{d\omega}\right)$ per $\alpha \omega \varphi$ exprimi, deinde α per $d\omega d\varphi$ eliminari posit, æquatio inter ω et φ obtinebitur, qvæ cursui lineæ perseguendo inserviet.

Singularis est casus, ubi sphærois per rotationem curvæ circa axem ipsius sphæroidis orta est, tum nimirum æqvalibus ϕ et p æqvalia competit V , unde seqvitur

$$\left(\frac{dV}{d\omega} \right) = 0$$

qvapropter æquatione (E) adducta

$$e \sin. \alpha \equiv \text{const.}$$

sive in puncto linea α initiali statuendo $e \equiv e'$ et $\alpha \equiv \alpha'$

$$e \sin. \alpha \equiv e' \sin. \alpha' \quad \dots \dots \quad (F)$$

sive sinus angulorum, qvibus linea brevissima meridianos intersecat, perpendicularibus, a punctis intersectionum in axem rotationis demissis, inverse sunt proportionales.

(Conferatur Monge: application de l'analyse, §. IV, des surfaces de révolution)

Proinde in casu considerato erit: P. 10. l. 8. v. 1.

$$d\omega = \tan \alpha \frac{P}{\xi} d\varphi = \frac{\xi' \sin \alpha' P}{\xi \xi' \cos \alpha'} \cdot d\varphi = \frac{\xi' P \sin \alpha'}{\xi \sqrt{\xi'^2 - \rho'^2 \sin^2 \alpha'/\xi'^2}} \cdot d\varphi \quad \dots \quad (G)$$

3.

Qvæ longe specialiora seqvuntur, omnia ad ellipsoidem spectant. Est æquatio ellipsis a centro notissima hæc:

$$y^2 = (1 - ee) (a^2 - x^2)$$

unde

$$\text{tang. } \varphi = -\frac{dx}{dy} = \frac{1}{x} \cdot \frac{\sqrt{a^2 - x^2}}{\sqrt{1 - ee}}$$

$$\begin{aligned} \text{tang. } \varphi &= -\frac{dx}{dy} = \frac{1}{x} \cdot \frac{\sqrt{a^2 - x^2}}{\sqrt{1 - ee}} \\ \ell' &= \frac{a \cos. \varphi}{\sqrt{1 - ee \sin. \varphi^2}} \end{aligned}$$

$$P = a (1 - ee) (1 - ee \sin. \varphi^2)^{-\frac{3}{2}}$$

$$\frac{P}{\rho} = \frac{1}{\cos. \varphi} \cdot \frac{1 - ee}{1 - ee \sin. \varphi^2}$$

$$\begin{aligned} P\rho &\equiv a^2 \cos. \varphi \cdot \frac{1 - ee}{(1 - ee \sin. \varphi^2)^{\frac{3}{2}}} \\ \sqrt{\rho^2 - \rho'^2 \sin. \alpha'^2} &\equiv a \cdot \left\{ \frac{\cos. \varphi^2 - \cos. \varphi'^2 \sin. \alpha'^2 - ee (\cos. \varphi^2 \sin. \varphi'^2 - \cos. \varphi'^2 \sin. \varphi^2 \sin. \alpha'^2)}{(1 - ee \sin. \varphi^2)^{\frac{1}{2}} (1 - ee \sin. \varphi'^2)^{\frac{1}{2}}} \right\}^{\frac{1}{2}} \end{aligned}$$

igitur

$$d\Sigma \equiv \frac{a (1 - ee) \cos. \varphi d\varphi (1 - ee \sin. \varphi^2)^{-\frac{3}{2}} (1 - ee \sin. \varphi'^2)^{\frac{1}{2}}}{\left\{ (\cos. \varphi^2 - \cos. \varphi'^2 \sin. \alpha'^2) (1 - ee) + ee \cos. \varphi^2 \cos. \varphi'^2 \cos. \alpha'^2 \right\}^{\frac{1}{2}}}$$

$$d\omega \equiv \frac{a (1 - ee) \cos. \varphi' d\varphi' (1 - ee \sin. \varphi'^2)^{\frac{1}{2}}}{\left\{ (\cos. \varphi^2 - \cos. \varphi'^2 \sin. \alpha'^2) (1 - ee) + ee \cos. \varphi^2 \cos. \varphi'^2 \cos. \alpha'^2 \right\}^{\frac{1}{2}}}$$

Qvæ ut facile integrantur, duplex admittenda est sumtio:

$$(1) \quad \sin. \varphi \equiv \sin. \varphi' \cos. \lambda + \cos. \varphi' \cos. \alpha' \sin. \lambda$$

$$\frac{\tan. \varphi'}{\cos. \alpha'} \equiv \tan. \gamma'$$

$$\sin. \varphi \equiv \frac{\sin. \varphi'}{\sin. \gamma'} \sin. (\gamma' + \lambda) \equiv \sin. \gamma \sin. \nu$$

tum erunt

$$\cos. \varphi'^2 \sin. \alpha'^2 \equiv \cos. \gamma^2$$

$$\cos. \varphi d\varphi \equiv \sin. \gamma \cos. \nu d\nu$$

$$1 - ee \sin. \varphi^2 \equiv 1 - ee \sin. \gamma^2 \sin. \nu^2$$

$$1 - ee \sin. \varphi'^2 \equiv 1 - ee \sin. \gamma^2 \sin. \nu'^2$$

$$\cos. \varphi^2 - \cos. \varphi'^2 \sin. \alpha'^2 \equiv \sin. \gamma^2 \cos. \nu^2$$

$$\cos. \phi^2 = 1 - \sin. \gamma^2 \sin. \nu^2$$

unde consequimur

$$d\Sigma = \frac{1}{\left\{ 1 + \frac{ee}{1-ee} \left(\sin y^2 \cos y^{1/2} + \cos y^2 \frac{\cos y^{1/2}}{\cos y^2} \right) \right\}^{1/2}} \quad \text{riga}$$

valorem ipsius $d\Sigma$ in seriem secundum potestates excentricitatis progredientem evolvendo, prodit neglectis biqvadratis superioribusqve potestatibus

$$d\Sigma = a\sqrt{1-ee} dy \left\{ 1 - ee \left(\frac{\sin \gamma^2}{2} - \frac{1}{2} \sin \gamma^2 \sin \nu^2 + \frac{1}{2} \cos \gamma^2 \frac{\cos \nu^2}{\cos \nu^2} \right) \right\}$$

quod integrale si $\alpha v = v'$ (sive $\alpha \equiv \alpha'$) sumatur.

$$\sum = a \sqrt{1 - ee} \left\{ (\nu - \nu') \left(1 + \frac{ee}{4} \sin^2 \gamma^2 \right) - \frac{e^2}{8} e^2 \sin^2 \gamma^2 (\sin 2\nu - \sin 2\nu') - \frac{e^2}{2} \cos \gamma^2 \left(\cos \nu'^2 \tan \nu - \frac{\sin 2\nu'}{2} \right) \right\}$$

$$= a \sqrt{1-e^2} \left\{ \lambda \left(1 + \frac{e^2}{4} \sin \gamma^2 \right) - \frac{3}{4} e^2 \sin \gamma^2 \cos (2\nu + \lambda) \sin \lambda - \frac{e^2}{2} \cos \gamma^2 \cos \nu' \frac{\sin \lambda}{\cos \nu} \right\}$$

$$\cot \alpha' \sin \delta \mp \sin \phi' \cos \delta$$

2) *andimentul tang.* $\theta = \arctan \frac{y}{x}$ unde $y = \sin \theta$ și $x = \cos \theta$.

mod. $\sin \theta \tan \mu' = \sin \theta \sin \phi'$ and vice versa.

tum erunt

$$\cos. \phi' \sin. \alpha' = \cos. \gamma$$

$$\frac{d\phi}{\sin \phi} = \tan \gamma \cos \mu \frac{du}{u}$$

$$\cos. \phi^2 = \frac{1}{1 + \tan^2 \mu^2 \sin^2 \mu^2}$$

$$1 - \frac{\cos. \phi'^2 \sin. \alpha'^2}{\cos. \phi^2} = \sin. \gamma^2 \cos. \mu^2$$

unde impetratur

$$d\omega = \frac{\sqrt{1-ee^2} \sin \gamma \cos \mu d\mu}{\left(1-ee^2 \sin^2 \phi^2\right)^{\frac{1}{2}}} + ee^2 \cos \phi d\phi$$

$$\text{II} \quad \int \frac{\sqrt{1 - ee^2} d\mu}{(1 - ee \sin \phi)^{1/2}} = \int \left(1 + \frac{ee^2 \cos \phi^{1/2} \cos \alpha^{1/2}}{1 - ee^2 \sin \gamma^2 \cos \mu^2} \right)^{1/2} d\mu$$

et simili modo explicando *dω* in seriem secundum potestates ipsius e progre-
dientem.

emergit

$$d\omega = d\mu \left\{ 1 - \frac{ee}{2} \left(\cos. \phi^{1/2} + \frac{\cos. \phi^{1/2} \cos. \alpha^{1/2}}{\sin. \gamma^2 \cos. \mu^2} \right) \right\}$$

Quum vero $d\phi$ nuper inventum sit $= \cos. \phi^2 \tan. \gamma \cos. \mu d\mu$ et idem $d\phi$

supra $= \frac{\sin. \gamma \cos. \nu dv}{\cos. \phi \cos. \mu}$; facile eruetur $d\mu \cdot \cos. \phi^2 = \cos. \gamma dv$; qvo substituto

habetur $d\omega = d\mu \left\{ 1 - \frac{ee}{2} \frac{\cos. \phi^{1/2} \cos. \mu^2}{\sin. \gamma^2 \cos. \mu^2} \right\} \frac{\cos. \gamma dv}{\cos. \nu}$

$$\omega = \mu - \frac{ee}{2} \frac{\cos. \phi^{1/2} \cos. \mu^2}{\sin. \gamma^2} \tan. \mu - \frac{ee}{2} \cos. \gamma v + \text{const.}$$

qvod integrale si $\alpha \phi = \phi'$, sive $v = v'$, sive $\mu = \mu'$ sumatur,

$$\begin{aligned} \omega &= \mu - \mu' - \frac{ee}{2} \frac{\cos. \phi^{1/2} \cos. \mu'^2}{\sin. \gamma^2 \cos. \mu'^2} (\tan. \mu - \tan. \mu') - \frac{ee}{2} \cos. \gamma (v - v') \\ &= \delta - \frac{ee}{2} \cos. \gamma \cdot \lambda - \frac{ee}{2} \cos. \phi^{1/2} \cos. \mu' \frac{\sin. \delta}{\cos. \mu'} \end{aligned}$$

Qui in formulis quantitatuum $\Sigma \omega$ obvii arcus, etsi, pro libero arbitrio, sive in partibus radii, sive in gradibus et graduum partibus exprimi possint, tamen est necesse, ut cum cæteris membris, quibus aggregatis definiuntur quantitatæ, homogenei reddantur; quapropter, si membra cætera in partibus radii exprimere jam decretum fuerit, ipsi arcus mensura eadem sunt exprimendi, si vero arcus in gradibus exprimere placuerit, membra cætera ad hanc ipsam mensuram per numerum notum reduci debent. Ut ambiguitati occurratur, quoties arcus in posterum simpliciter designentur, ad mensuram graduum intelligentur respicere.

4.

Jam igitur æquationes conditionales colligendo

$$\cos. \gamma = \sin. \alpha' \cos. \phi' \quad \text{ad calc. confirm.} \quad \sin. \gamma = \frac{\sin. \phi'}{\sin. \nu'}$$

$$\tan. \mu' = \tan. \alpha' \sin. \phi' \quad \sin. \mu' = \frac{\tan. \phi'}{\tan. \gamma}$$

$$\tan. \nu' = \frac{\tan. \phi'}{\cos. \alpha'} \quad \cos. \nu' = \frac{\cos. \alpha' \cos. \phi'}{\sin. \gamma}$$

$$\sin. \mu = \frac{\tan. \phi}{\tan. \gamma} \quad \tan. \mu = \cos. \gamma \tan. \nu$$

sin. $\nu = \frac{\sin. \phi}{\sin. \gamma}$) ienes aequalibus tang. $(45^\circ - \frac{\nu}{\gamma}) = \sqrt{\tan(\frac{\nu}{\gamma}) \cot(\frac{\nu + \phi}{2})}$
 -micasa mico si $\nu < 45^\circ$ tang. $(45^\circ - \frac{\nu}{\gamma}) > \sqrt{\tan(\frac{\nu}{\gamma}) \cot(\frac{\nu + \phi}{2})}$ \therefore $\sin. \delta = \sin. \alpha' \frac{\sin. \lambda}{\cos. \phi}$
 $\lambda = \nu - \nu'$; $\delta = \mu - \mu'$ \therefore $\sin. \delta = \sin. \alpha' \frac{\sin. \lambda}{\cos. \phi}$
 oritur

$$\omega = \delta - \frac{e^2}{2} \cos. \gamma. \lambda - \frac{e^2}{2} \cos. \phi'^{1/2} \cos. \mu' \frac{\sin. \delta}{\sin. \mu} \quad (1. a)$$

formula, qva ω ex $\phi' \alpha' \phi$ obtenetur (1. a)

Manifestum est, si α pro α' ponatur, ϕ' cum ϕ permutetur, eadem via et ratione ω ex $\phi' \alpha \phi$ erui posse (1. b)

Retinendo:

cos. $\gamma = \sin. \alpha' \cos. \phi'$ \therefore $\cos. \gamma = \frac{\sin. \phi'}{\sin. \nu}$

$$\tang. \nu' = \frac{\tang. \phi'}{\cos. \alpha'} \quad \cos. \nu' = \frac{\cos. \omega}{\sin. \gamma} \quad (2. a)$$

$$\tang. \mu' = \tang. \alpha' \sin. \phi' \quad \sin. \mu' = \tang. \phi' \cot. \gamma$$

et introducendo $\delta = \omega + \frac{e^2}{2} \cos. \gamma (\nu - \nu')$ (2)

$$\tang. N = \tang. \frac{(\phi + \mu')}{\cos. \gamma} \quad (2. b)$$

conversione simplici oritur: $\mu' = \delta + \mu$

$$\delta = \omega + \frac{e^2}{2} \cos. \gamma (\nu - \nu') + \frac{e^2}{2} \frac{\cos. \phi'^{1/2}}{\sin. \nu'} \cdot \frac{\cos. \mu'}{\cos. (\omega + \mu')} \cdot \sin. \omega \quad (2)$$

unde, quum ω ex $\phi' \alpha' \phi$ erui posse

$$\mu' = \delta + \mu \quad \tang. \phi' = \sin. \mu' \tang. \gamma \quad (2. c)$$

ϕ ex $\phi' \alpha' \omega$ erui potest (2. a)

Qvodsi ipsi α' sufficiatur α et ϕ' cum ϕ permutetur, via eadam ex $\phi \alpha \omega$ prodit valor ϕ' (2. b)

Qvinetiam series (2) valori α' ex $\phi' \phi \omega$ datis eruendo inserviet, habetur enim ex art. 3

$$\cot. \alpha' = \frac{\tang. \phi \cos. \phi' - \sin. \phi' \cos. \delta}{\sin. \delta} \quad (2. d)$$

sive introducendo $\tang. \phi = \frac{\tang. \phi}{\cos. \delta}$

$$\cot. \alpha' = \frac{\cot. \delta}{\sin. (\phi - \phi')} = \frac{\tang. \phi \sin. (\phi - \phi')}{\sin. \phi \sin. \delta} \quad (2. e)$$

proptereal in aequatione hac (supponendo) $\delta = \omega$, valor approximatus α' erui po-

test, qvo si computentur æqvationes conditionales seriei (2) una cum ipsa serie, correctus obtinetur valor δ . Correcto igitur valore δ in locum suppositi substituto, item correctus computabitur valor α' (2. c)

Similiter ϕ' ex $\alpha' \phi \omega$ datis vi ejusdem seriei (2) eruetur:

Scilicet in art. 3 statuitur

$$\text{tang. } \phi = \frac{\cos. \phi'}{\cos. \phi}$$

unde introducto tang. $\phi = \cos. \delta \cot. \phi$

seqvitur $\cos. \phi' + \phi = \tan. \delta \cot. \alpha' \sin. \phi = \sin. \delta \cot. \alpha' \cos. \phi = \cot. \phi$
qvapropter, $\delta = \omega$ supponentes, per æqvationem allatam $\phi' + \phi$, adeoque ϕ' approximate nanciscemur, porro ϕ' approximato æqvationes conditionales seriei (2) et ipsam seriem computabimus, unde correctus valor δ . Ex correcto δ correctum ϕ' per æqvationem allatam emerget (2. d)

Collectis æqvationibus conditionalibus:

$$\cos. \gamma = \sin. \alpha' \cos. \phi' \quad \text{ad confirm. calc.}$$

$$\sin. \gamma = \frac{\sin. \phi'}{\sin. \nu}$$

$$\text{tang. } \nu' = \frac{\tan. \phi'}{\cos. \alpha'}$$

$$\cos. \nu' = \frac{\cos. \alpha' \cos. \phi'}{\sin. \gamma}$$

$$\sin. \nu = \frac{\sin. \phi}{\sin. \gamma} \quad \tan. \left(45^\circ - \frac{\nu}{2}\right) = \sqrt{\tan. \left(\frac{\gamma - \phi}{2}\right) \cot. \left(\frac{\gamma + \phi}{2}\right)}$$

$$\lambda = \nu - \nu' = \left(45^\circ - \frac{\nu}{2}\right) - \left(45^\circ - \frac{\gamma - \phi}{2}\right) = \frac{\gamma - \phi}{2}$$

habetur

$$\Sigma = a \sqrt{1 - ee} \left[\left(1 + \frac{e^2}{4} \sin. \gamma^2 \right) \lambda \sin. 1'' - \frac{3}{4} e^2 \sin. \gamma^2 \cos. (2\nu' + \lambda) \sin. \lambda - \frac{e^2}{2} \cos. \gamma^2 \cos. \nu' \frac{\sin. \lambda}{\cos. \nu} \right]$$

formula, qva Σ ex $\phi' \alpha' \phi$ obtinetur (3. a)

Ponendo α pro α' et ϕ' cum ϕ permutando, Σ ex $\phi' \alpha' \phi$ pariter consequemur (3. b)

Retentis æqvationibus conditionalibus:

$$\cos. \gamma = \sin. \alpha' \cos. \phi'; \quad \text{ad calc. confirm. } \sin. \gamma = \frac{\sin. \phi'}{\sin. \nu}$$

$$\tan. \nu' = \frac{\tan. \phi'}{\cos. \alpha'} \quad \cos. \nu' = \frac{\cos. \alpha' \cos. \phi'}{\sin. \gamma}$$

$$\text{et introducto } \sigma = \frac{\Sigma}{a \sqrt{1 - ee}} \cdot \sin. 1'' \quad \text{et } \sigma = \frac{\Sigma}{a \sqrt{1 - ee}} \cdot \sin. 1''$$

per simplicem conversionem prodit

$$\lambda = \sigma \left(1 + \frac{e^2}{4} \sin. \gamma^2 \right) + \frac{3}{4} \frac{ee}{\sin. 1''} \sin. \gamma^2 \cos. (2\nu' + \sigma) \sin. \sigma + \frac{e^2 \cos. \gamma^2}{2 \sin. 1'' \cos. (\sigma + \nu')} \cos. \nu' \sin. \sigma . . . (4)$$

unde, quum

$$\nu = \lambda + \nu'$$

$$\sin. \phi = \sin. \nu \sin. \nu'$$

ϕ ex ϕ' α' Σ datis erui potest (4. a)

Si α surrogetur ipsi α' et ϕ' cum ϕ permutetur, ratione simili ϕ' ex ϕ α Σ nobis sese obferet (4. b)

Ope ejusdem seriei (4) α' ex ϕ' ϕ Σ datis obtinebimus,
quum enim juxta art. 3

$$\cos. \alpha' = \frac{\sin. \phi - \sin. \phi' \cos. \lambda}{\cos. \phi' \sin. \lambda}$$

sive, inferendo

$$\tan. \zeta = \frac{\cos. \lambda \cos. \phi'}{\sin. \phi}$$

$$\cos. \alpha' = \frac{\sin. (\phi - \zeta)}{\cos. \zeta \cos. \phi' \sin. \lambda} = \frac{\tan. \phi' \cot. \lambda}{\sin. \zeta \cos. \phi} \cdot \sin. (\phi - \zeta)$$

supponentes $\lambda = \sigma$, valorem approximatum α' obtinebimus, qvo invento, æquationes conditionales seriei (4) et ipsam seriem computabimus, unde correctus valor λ . Correctum λ in locum suppositi surrogantes, correctum α' investigabimus . (4. c)

Ipsius ejudem seriei adminiculo ϕ' ex ϕ Σ α' lucrabimur, qvod ut contingat, ex art. 3 adducetur æquatio :

$$\sin. \phi = \sin. \phi' \cos. \lambda + \cos. \phi' \sin. \lambda \cos. \alpha'$$

sive, admissio $\tan. \tau = \tan. \lambda \cos. \alpha'$

$$\sin. (\phi' + \tau) = \frac{\cos. \tau}{\cos. \lambda} \sin. \phi = \frac{\sin. \tau \sin. \phi}{\sin. \lambda \cos. \alpha'}$$

porro supponetur $\lambda = \sigma$, et valor approximatus $\phi' + \tau$ per æquationem allatam eruetur, unde habebitur ϕ' . Cujus qualicunque valore si æquationes seriei (4) una cum ipsa serie computentur, correctum sistetur λ , qvod in locum suppositi surrogatum, calculo simili continuato, ad ipsum correctum ϕ' adducet . . (4. d)

5.

Duplex est casus, ubi formulæ jam expositæ insignem patiuntur contractio-
nem. Alter, si $\alpha' = 90^\circ$ et linea meridiano perpendicularis, alter, si $\alpha' = 0$,
adeo ut tota linea in meridianum incidat. De singulis singulatim dicendum.

Ubi $\alpha' = 90^\circ$, si linea brevisima eademque perpendicularis designetur Σ , latitudo puncti, in quo perpendicularis meridianum ferit, $\bar{\phi}$, transbit γ in $\bar{\phi}$, ν' in 90° , μ' in 90° , si porro, ne signorum multitudine obruamur, quantitates ν et μ his adstrictas conditionibus, per signa analogae $\bar{\nu}$ et $\bar{\mu}$ denotemus, prodibunt:

$$\begin{aligned}\sin. \nu &= \frac{\sin. \phi}{\sin. \bar{\phi}} & \tan. \left(45^\circ - \frac{\nu}{2}\right) &= \pm \sqrt{\tan. \left(\frac{\bar{\phi} - \phi}{2}\right) \cot. \left(\frac{\bar{\phi} + \phi}{2}\right)} \\ \sin. \bar{\mu} &= \frac{\tan. \phi}{\tan. \bar{\phi}} & \tan. \bar{\mu} &= \cos. \bar{\phi} \tan. \nu\end{aligned}$$

gvare,

si dicatur

$$\bar{\lambda} = \bar{\nu} - 90^\circ \quad \bar{\sigma} = \bar{\mu} - 90^\circ$$

æquationes conditionales oriuntur:

$$\cos. \delta = \frac{\tan. \phi}{\tan. \lambda} ; \quad \text{ad calc. conf. } \tan. \delta = \frac{\tan. \lambda}{\cos. \phi}$$

$$\cos. \bar{\lambda} = \frac{\sin. \phi}{\sin. \bar{\phi}} ; \quad \text{tang. } \frac{\bar{\lambda}}{2} = \pm \sqrt{\text{tang.} \left(\frac{\bar{\phi}-\phi}{2} \right) \cot. \left(\frac{\bar{\phi}+\phi}{2} \right)}$$

unde

$$\omega = \bar{\delta} - \frac{1}{2} ee \cos \varphi + \lambda$$

Hinc per conversionem,

introducto tang. $\Lambda \equiv \tan. \omega \cos. \phi$

oritur

$$\bar{\delta} = \omega + \frac{1}{2} ee \cos. \varphi, \bar{\Lambda}$$

quare, quum

$$\text{tang. } \varphi = \cos. \delta \text{ tang. } \varphi$$

Similiter, inferendo

$$\text{tang. } \tilde{\Phi} = \frac{\text{tang. } \phi}{\cos \omega}$$

$$\text{tang. } \bar{\varrho} \equiv \text{tang. } \omega \cos. \bar{\Phi} \quad ; \quad \cos. \bar{\varrho} \equiv \frac{\sin. \bar{\Phi}}{\sin. \bar{\Phi}}$$

comparat

$$\omega + \frac{1}{\epsilon} ee \cos. \Phi$$

$$\text{unde, per æqvat. } \tan \phi = \frac{\tan \alpha}{\cos \beta}$$

Repetita æquatione

$$\cos \lambda = \frac{\sin \phi}{\sin \bar{\phi}} ; \quad \text{sive} \tan \frac{\lambda}{2} = \pm \sqrt{\tan \left(\frac{\bar{\phi} - \phi}{2} \right) \cot \left(\frac{\bar{\phi} + \phi}{2} \right)}$$

prodit

$$\bar{\Sigma} = a \sqrt{1-e^2} \left\{ \left(1 + \frac{e^2}{4} \sin^2 \bar{\phi} \right) \bar{\lambda} \sin \bar{\iota}'' + \frac{3}{4} e^2 \sin \bar{\phi}^2 \sin \bar{\lambda} \cos \bar{\lambda} \right\}$$

Ponendo

$$\sigma_i = \frac{M_i}{a\sqrt{1-e_i} \cdot \sin_i 1^\circ}$$

~~existit~~

$$\bar{\lambda} = \bar{\sigma} \left(1 - \frac{e^2}{4} \sin \bar{\phi}^2 \right) - \frac{3}{4} \frac{e^2}{\sin \bar{\phi}^2} \sin \bar{\phi}^2 \sin \bar{\sigma} \cos \bar{\sigma}$$

unde, adducta æqvat. $\sin. \phi = \sin. \bar{\phi} \cos. \lambda$

Itidem, statuendo

$$\sigma = \frac{M}{a\sqrt{1-e^2} \cdot \sin i}$$

$$\sin \gamma = \frac{\sin \phi}{\cos \alpha}$$

erit

$$\bar{\lambda} = \frac{1}{\sigma} \left\{ 1 - \frac{e^2}{4} \sin \bar{\mathfrak{F}}^2 \right\} - \frac{3}{4} \frac{e^2}{\sin \bar{\mathfrak{F}}^2} \sin \bar{\mathfrak{F}}^2 \sin \sigma \cos \sigma$$

aware *swim*

$$\sin \bar{\phi} = \frac{\sin \phi}{\cos \alpha}$$

Ubi $\alpha' = 0$, si tum linea brevisima, tota in meridianum incidens, notetur Σ , latitudo puncti, quo incipit, φ' , latitudo puncti, quo desinit, $\bar{\varphi}$, transformabitur γ in 90° , ν' in φ' , ν in φ et $\bar{\varphi}$ in $\bar{\varphi}$, itaque,

adsumendo $\lambda = \tilde{\phi} - \phi$

$$\sum = a \sqrt{1-e^2} \left\{ \lambda \sin. i'' \left(i + \frac{e^2}{4} \right) - \frac{3}{4} e^2 \sin. \lambda \cos. (\bar{\phi} + \phi') \right\}$$

$$\text{Perhibendo } \sigma = \frac{\sum}{a\sqrt{1-ee} \sin. i''}$$

$$\lambda = \sigma \left(x - \frac{e^2}{4} \right) + \frac{3}{4} \frac{e^2}{\sin. \pi''} \sin. \sigma \cos. (\sigma + 2\varphi')$$

qvapropter, quum $\bar{\varphi} = \lambda + \varphi'$

Similiter, sistendo

$$\sigma_i = \frac{\sum}{a\sqrt{1-e_e} \sin i''}$$

$$\lambda = \sigma \left(1 - \frac{e^2}{4} \right) + \frac{3}{4} \frac{e^2}{\sin. i''} \sin. \sigma \cos. (2\bar{\phi} - \sigma)$$

unde, propter æqv. $\phi' = \bar{\phi} - \lambda$

Ut formulæ, qvarum freqventissimus sit usus, in numeros currentes referantur, jam seqventes sunt adjungendæ.

Quum juxta æquationem (F) habetur $\varrho \sin. \alpha = \varrho' \sin. \alpha'$

sive propter art. 3

$$\frac{\cos \phi \sin \alpha}{\sqrt{1 - ee \sin^2 \phi}} = \frac{\cos \phi' \sin \alpha'}{\sqrt{1 - ee \sin^2 \phi'^2}}$$

commode adducentur

$$\text{tang. } f \equiv \sqrt{1-e_e} \text{ tang. } \varphi, \quad \text{tang. } f' \equiv \sqrt{1-e_e} \text{ tang. } \varphi'$$

$$\text{unde } \cos. f \sin. \alpha \equiv \cos. f' \sin. \alpha'$$

formula, qva, qvantitatum $\varphi \propto \varphi' \alpha'$ tribus datis, qvarta eruetur (14)

Specialis est casus, ubi $\alpha' \equiv 90^\circ$, tum enim φ' transibit in $\bar{\varphi}$ et α in $\bar{\alpha}$, qvapropter, introducendo $\tan f = \sqrt{1 - ee} \tan \varphi$; $\tan \bar{f} = \sqrt{1 - ee} \tan \bar{\varphi}$ obtinetur $\cos f \sin \alpha \equiv \cos \bar{f}$ formula, qva, qvantitatum $\varphi \bar{\varphi}$ duabus datis, tertia indagatur (15)

Uti hactenus suppositum est lineas Σ et $\bar{\Sigma}$ ab uno aliquo punto per φ et ω determinato, meridianum versus, in quem tota incidat linea $\dot{\Sigma}$, excurrere, ita manifestum erit, quod si angulus, quo altera ab altera in puncto dicto deflectat, dicatur ψ , fore

Quinetiam lineæ Σ et $\bar{\Sigma}$ mutuo se intersecantes, ipsam tertiam intersecant lineam Σ , unde gignitur triangulum, cui considerando articuli seqentes dicati sint.

6. Casus triangula meridiano-rectangula

In triangulo ellipsoidico, duplii adstricto conditioni, ut angulum habeat rectum et duo puncta angularia in meridiano eodem sita, designentur: latitudines primi, secundi et tertii puncti angularis φ' $\bar{\varphi}$ φ latera, sive lineæ brevissimæ inter primum et secundum, secundum et tertium, primum et tertium punctum angulare in superficie ellipsoidis ductæ Σ $\bar{\Sigma}$ Σ anguli prioribus, rectum comprehendentibus, oppositi, ad ordinem ψ α differentia longitudinis, qva et primum et secundum punctum angulare, utrumque in meridiano eodem situm, a tertio discrepat ω tum seqentes oriuntur casus, in quibus tria elementa cæteris inveniendis sufficiunt.

Quum autem primario fini, quo talia triangula meridiano-rectangula (sit vena voci) solvuntur, jam erit satisfactum, si modo $\varphi' \varphi \Sigma \alpha' \omega$ innotescant, brevitas ratio, ut in his solis investigandis subsistamus, valde svadet.

1. Datis $\varphi' \varphi \omega$

Ex $\varphi' \varphi \omega$ obtinetur α' juxta form. 2.c; ex $\varphi' \alpha' \varphi$ obtinetur Σ juxta form. 3.a.

2. Datis $\varphi' \bar{\varphi} \omega$

Ex $\bar{\varphi} \varphi \omega$ obtinetur α' juxta form. 5; unde casus hic ad 1 reducitur.

3. Datis $\varphi' \bar{\varphi} \omega$

Ex $\bar{\varphi} \omega$ obtinetur φ juxta form. 6; unde casus hic ad 1 reducitur.

4. Datis $\varphi' \bar{\varphi} \bar{\Sigma}$

Ex $\bar{\varphi} \bar{\Sigma}$ obtinetur φ juxta form. 9; unde casus hic ad 2 reducitur.

5. Datis $\varphi' \varphi \bar{\Sigma}$

Ex $\varphi \bar{\Sigma}$ obtinetur $\bar{\varphi}$ juxta form. 10; unde casus hic ad 2 reducitur.

6. Datis $\varphi' \varphi \bar{\Sigma}$

Ex $\varphi' \bar{\Sigma}$ obtinetur $\bar{\varphi}$ juxta form. 12; unde casus hic ad 2 reducitur.

7. Datis $\phi' \Sigma \bar{\Sigma}$
Ex $\phi' \Sigma$ obtinetur $\bar{\phi}$ juxta form. 12; unde casus hic ad 4 reducitur.
8. Datis $\phi' \omega \bar{\Sigma}$
Ex $\phi' \bar{\Sigma}$ obtinetur $\bar{\phi}$ juxta form. 12; unde casus hic ad 3 reducitur.
9. Datis $\bar{\phi} \phi \bar{\Sigma}$
Ex $\bar{\phi} \Sigma$ obtinetur ϕ' juxta form. 13; unde casus hic ad 2 reducitur.
10. Datis $\bar{\phi} \omega \bar{\Sigma}$
Ex $\bar{\phi} \bar{\Sigma}$ obtinetur ϕ' juxta form. 13; unde casus hic ad 3 reducitur.
11. Datis $\bar{\phi} \Sigma \bar{\Sigma}$
Ex $\bar{\phi} \bar{\Sigma}$ obtinetur ϕ' juxta form. 13; unde casus hic ad 4 reducitur.
12. Datis $\phi \omega \bar{\Sigma}$
Ex $\phi \omega$ obtinetur $\bar{\phi}$ juxta form. 7; ex $\phi \Sigma$ obtinetur ϕ' juxta form. 13; unde casus hic ad 1 reducitur.
13. Datis $\phi \bar{\Sigma} \bar{\Sigma}$
Ex $\phi \bar{\Sigma}$ obtinetur $\bar{\phi}$ juxta form. 10; ex $\bar{\phi} \bar{\Sigma}$ obtinetur ϕ' juxta form. 13; unde casus hic ad 2 reducitur.
14. Datis $\phi \omega \alpha'$
Ex $\phi \omega \alpha'$ obtinetur ϕ' juxta form. 2. d.; ex $\phi' \alpha' \phi$ obtinetur Σ juxta form. 3. a.
15. Datis $\bar{\phi} \phi \alpha'$
Ex $\bar{\phi} \phi$ obtinetur ω juxta form. 5; unde casus hic ad 14 reducitur.
16. Datis $\bar{\phi} \omega \alpha'$
Ex $\bar{\phi} \omega$ obtinetur ϕ juxta form. 6; unde casus hic ad 14 reducitur.
17. Datis $\bar{\phi} \alpha' \bar{\Sigma}$
Ex $\bar{\phi} \bar{\Sigma}$ obtinetur ϕ juxta form. 9; unde casus hic ad 15 reducitur.
18. Datis $\phi \alpha' \bar{\Sigma}$
Ex $\phi \bar{\Sigma}$ obtinetur $\bar{\phi}$ juxta form. 10; unde casus hic ad 15 reducitur.
19. Datis $\phi' \phi \alpha'$
Ex $\phi' \phi \alpha'$ obtinetur $\omega \Sigma$ juxta formm. 1. a., 3. a.

20. Datis ϕ' , ω , α' obtinetur ϕ ex ϕ' , ω , α' juxta form. 2. a; Σ ex ϕ' , ϕ , α' juxta form. 3. a.
21. Datis ϕ' , ϕ , Σ obtinetur α' ex ϕ' , ϕ , Σ juxta form. 4. c; ω ex ϕ' , ϕ , α' juxta form. 1. a.
22. Datis ϕ' , α' , Σ obtinetur ϕ ex ϕ' , α' , Σ juxta form. 4. a; ω ex ϕ' , ϕ , α' juxta form. 1. a.
23. Datis ϕ , α' , Σ obtinetur ϕ' ex ϕ , α' , Σ juxta form. 4. d; ω ex ϕ' , α' , ϕ juxta form. 1. a.
24. Datis $\bar{\phi}$, ϕ , ψ obtinetur ω ex $\bar{\phi}$, ϕ , ψ juxta form. 5. a; α ex $\bar{\phi}$, ψ juxta form. 16; ϕ' ex ϕ , ω juxta form. 2. b; α' ex ϕ' , ϕ , α juxta form. 14; Σ ex ϕ' , α' , ϕ juxta form. 3. a.
25. Datis ϕ , Σ , ψ obtinetur $\bar{\phi}$ ex ϕ , Σ , ψ juxta form. 10; unde casus hic ad 24 reducitur.
26. Datis $\bar{\phi}$, Σ , ψ obtinetur ϕ ex $\bar{\phi}$, Σ , ψ juxta form. 9; unde casus hic ad 24 reducitur.
27. Datis $\bar{\phi}$, ω , ψ obtinetur ϕ ex $\bar{\phi}$, ω , ψ juxta form. 6; α ex $\bar{\phi}$, ϕ obtinetur α juxta form. 15 et sic porro ut supra in casu 24.
28. Datis ϕ , ω , ψ obtinetur $\bar{\phi}$ ex ϕ , ω , ψ juxta form. 7; α ex $\bar{\phi}$, ϕ juxta form. 15, et sic porro ut supra in casu 24.

Qui subseqvuntur casus, licet tria data elementa triangulo determinando sufficientia contineant, directa tamen via solvi neqveunt. Quum autem cumullo qvopiam jam soluto casu duo communia habeant elementa, tota res in eo cardine verti cernitur, ut illius jam soluti casus, qvi cum proposito duo habeat elementa communia, peculiare elementum inveniatur. Hinc methodus indirecta sese offert hæc. Supponatur arbitrarius hujus desiderati elementi valor, qvo duobus datis

as sociato, elementum tertium juxta præcepta art. præcedentis investigari qveat. Utrum verus necne fuerit iste desiderati elementi valor, comparatione inter computatum et datum tertii elementi valorem instituta decernitur. Si enim conveniunt, valor desiderati elementi exactus censebitur; si discrepant, alium tentare jubemur desiderati elementi valorem, ex qvo forte computatus qvidam tertii elementi valor, a dato haud diversus; proficiscatur, et haec porro iterare tentamina, donec id revera contingat. Duo sunt, qvæ qvoad methodum hanc optanda relinquuntur: alterum, qvod suppositus desiderati elementi valor a veritate haud nimis recedit, alterum, qvod, dato cum computato tertii elementi valore comparando, non solum, an ulla suppositi aberratio obtineat, verum etiam qvanta, proxime saltem docemur. Conditione excentricitatis perparvæ, qvam semper statuimus, manente, utriqve qvodammodo erit satisfactum; priori, si, triangulo sphæroidico tanquam sphærico spectato, elementum desideratum juxta præcepta trigonometriæ sphæricæ investigetur; posteriori, si in omnibus æquationibus, per qvas a supposito elementi desiderati valore ad tertium usqve elementum descendimus, illud ratione hujus, sive qvod idem est, hoc ratione illius differentietur. Haud disimulandum est, hanc differentiationem plurijm æquationum, pluribus constantium membris, molestissimam fore; nihil vero impedit, qvo minus labor iste admodum minui qveat. Qvo enim jure differentiam inter datum et computatum tertii elementi valorem tanquam differentiale spectare licebit, eodem, haud minori, membra æquationum, potestatibus excentricitatis affecta, audebimus inter differentiandum negligere. Si illa igitur per differentiationem inventa et in suppositum elementi desiderati valorem adhibenda correctio errori, ex dupli hoc fonte emananti, obnoxia videretur, — id qvod revera accidit, qvoties triangulum haud mediocre est, — tum in valore elementi desiderati ita correcto haud acqviescemos, at potius novam aggrediemur hypothesin, correcto valore ad correctiorem inveniendum eadem via et ratione utentes, qva supposito ad correctum deprehendendum antea usi sumus.

29. Datis ϕ' $\bar{\phi}$ α' Supponatur ω ex $\phi' \bar{\phi} \alpha'$ per trig. sphæric. deductum; obtinetur ω ex $\phi' \alpha' \bar{\phi}$ juxta form. 1. a; $\bar{\phi}$ ex $\phi \omega$ juxta form. 7; notatur $d\bar{\phi}$ differentia inter $\bar{\phi}$ da-

tum et comput.; differentiando $\bar{\phi}$ ratione ϕ obtinetur variatio $d\phi$; ϕ hac correctum meliorem constituet hypothesin, qva idem cursus repeti potest. Unde casus hic ad 19 reducetur.

Aliter

Supponatur ϕ ex $\phi' \alpha' \bar{\phi}$ per trig. sphæric. deductum; ex $\bar{\phi} \phi$ obtinentur $\omega \bar{\Sigma} \alpha$ juxta formm. 5, 8, 15; ex $\phi' \alpha' \omega$ obtinetur juxta form. 2. a aliud ϕ insigniendum (ϕ); notatur $d\phi$ differentia inter ϕ et (ϕ); ex $\phi' \alpha' (\phi)$, invenitur α juxta form. 14; ex $d\phi \alpha \alpha$ obtinetur $d\bar{\Sigma}$ per trig. sphæric.; qva correctione in $\bar{\Sigma}$ adhibita prodit ($\bar{\Sigma}$); ex $\bar{\phi} (\bar{\Sigma})$, obtinetur juxta form. 9 valor ϕ veritati proximus, qvo si idem cursus unica vice repetatur, exactus prodibit. Unde casus hic ad 19 reducetur.

30. Datis $\phi' \bar{\Sigma} \alpha'$

Ex $\phi' \bar{\Sigma}$ obtinetur $\bar{\phi}$ juxta form. 12; unde casus hic ad 29 reducetur.

31. Datis $\bar{\phi} \alpha' \bar{\Sigma}$

Ex $\bar{\phi} \bar{\Sigma}$ obtinetur ϕ' juxta form. 13; unde casus hic ad 29 reducetur.

32. Datis $\phi' \bar{\phi} \bar{\Sigma}$

Supponatur α' ex $\phi' \bar{\phi} \bar{\Sigma}$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi' \alpha' \bar{\Sigma}$ juxta form. 4. a; ω ex $\phi' \alpha' \phi$ juxta form. 1. a; $\bar{\phi}$ ex $\phi \omega$ juxta form. 7; notatur $d\bar{\phi}$, differentia inter $\bar{\phi}$ datum et comput.; differentiando $\bar{\phi}$ ratione α' obtinetur variatio $d\alpha'$; α' hac correctum meliorem constituet hypothesin, qva idem cursus repeti potest. Unde casus hic ad 22 reducetur.

Aliter

Supponatur ϕ ex $\phi' \bar{\phi} \bar{\Sigma}$ per trig. sphæric. deductum; ex $\bar{\phi} \phi$ obtinentur $\omega \bar{\Sigma} \alpha$ juxta formm. 5, 8, 15; obtinetur α' ex $\phi' \phi \omega$ juxta form. 2. c; ex $\phi' \alpha' \phi$ obtinentur $\bar{\Sigma} \alpha$ juxta formm. 3. a, 14; ex $\alpha \alpha$ obtinetur ψ juxta form. 16; notatur $d\Sigma$ differentia inter Σ datum et comput.; solvendo triang. sphæric. rectang. qvod determinatur per $d\Sigma$ et ($\psi - 90^\circ$) obtinetur $d\bar{\Sigma}$, qva correctione in $\bar{\Sigma}$ adhibita prodit ($\bar{\Sigma}$); ex $\bar{\phi} (\bar{\Sigma})$, prodit per form. 9 valor ϕ veritati proximus, qvo si idem cursus unica vice repetatur, exactus sistetur. Unde casus hic ad 21 reducetur.

33. Datis $\phi' \Sigma \bar{\Sigma}$

Ex $\phi' \Sigma$ obtinetur $\bar{\phi}$ juxta form. 12; unde casus hic ad 32 reducitur.

34. Datis $\bar{\phi} \Sigma \bar{\Sigma}$

Ex $\bar{\phi} \Sigma$ obtinetur ϕ' juxta form. 13; unde casus hic ad 32 reducitur.

35. Datis $\phi' \bar{\phi} \psi$

Supponatur ϕ ex $\phi' \bar{\phi} \psi$ per trig. sphæric. deductum; ex $\bar{\phi} \phi$ obtinentur $\omega \alpha$ per formulas 5, 15; obtinetur α ex $\alpha \psi$ juxta form. 16; ϕ' ex $\phi \omega \alpha$ juxta form. 2. b; notatur $d\phi'$, differentia inter ϕ' datum et comput.; differentiando ϕ' ratione ϕ obtinetur variatio $d\phi$; ϕ hac correctum meliorem constituet hypothesis, qva idem cursus repeti potest. Vero ϕ invento eodemque cursu, donec verum α asseqvamur, continuato, ex $\phi' \phi \alpha$ obtinentur α' juxta formm. 14, 3. b.

Aliter

Supponatur ϕ ex $\phi' \bar{\phi} \psi$ per trig. sphæric. deductum; ex $\bar{\phi} \phi$ obtinentur $\omega \bar{\Sigma} \alpha$ juxta formm. 5, 8, 15; ex $\phi \omega \phi'$ obtinetur α' juxta form. 2. c; α ex $\phi' \alpha' \phi$ juxta form. 14; (ψ) ex $\alpha \bar{\alpha}$ juxta form. 16 computari potest; solvendo triang. sphæric. rectang. qvod determinatur per $(180^\circ - \psi)$ et $(\psi, -90^\circ)$ obtinetur $d\bar{\Sigma}$, qva correctione in $\bar{\Sigma}$ adhibita prodit $\bar{\Sigma}$; ex $\bar{\phi} \bar{\Sigma}$, obtinetur juxta form. 9 valor ϕ veritati proximus, qvo si idem cursus unica vice repeatatur, exactus prodibit. Vero ϕ invento, et sic porro uti in solutione antecedente.

36. Datis $\phi' \Sigma \psi$

Ex $\phi' \Sigma$ obtinetur $\bar{\phi}$ juxta form. 12; unde casus hic ad 35 reducitur.

37. Datis $\bar{\phi} \Sigma \psi$

Ex $\bar{\phi} \Sigma$ obtinetur ϕ' juxta form. 13; unde casus hic ad 35 reducitur.

38. Datis $\phi' \omega \Sigma$

Supponatur α' ex $\phi' \omega \Sigma$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi' \alpha' \omega$ juxta form. 2. a; (Σ) , ex $\phi' \alpha' \phi$ juxta form. 3. a; notatur $d\Sigma$ differentia inter Σ datum et comput.; differentiando Σ ratione α' obtinetur variatio $d\alpha'$, qva α' corrigi potest et correcto α' idem cursus repeti.

Aliter

Supponatur α' ex ϕ' ω Σ per trig. sphæric. deductum; obtinetur ϕ ex ϕ' α' ω juxta form. 2. a; obtinentur Σ , α ex ϕ' α' ϕ juxta formm. 3. a, 14; notatur $d\Sigma$, differentia inter Σ datum et comput.; solvendo triang. sphæric. rectang., qvod determinatur per $d\Sigma$ et $(\alpha - 90^\circ)$, obtinetur variatio arcus $d\alpha'$; α' hac correctum veritati tantum accedit, ut cursu eodem semel repetito, ipsum verum sistatur.

39. Datis ϕ ω Σ

Supponatur α ex ϕ ω Σ per trig. sphæric. deductum; ex ϕ ω α obtinetur ϕ' juxta form. 2. b; Σ ex ϕ' ϕ α juxta form. 3. b; notatur $d\Sigma$, differentia inter Σ datum et comput.; differentiando Σ ratione α obtinetur variatio $d\alpha$; α hac correctum meliorem constituet hypothesin, qva idem cursus repeti potest. Vero α invento et vero ϕ' , ex ϕ' ϕ α obtinetur α' juxta form. 14.

Aliter

Supponatur α ex ϕ ω Σ per trig. sphæric. deductum; ex ϕ ω α obtinetur ϕ' juxta form. 2. b; ex ϕ α ϕ' obtinentur Σ α' juxta formm. 3. b, 14; notatur $d\Sigma$, differentia inter Σ datum et comput.; solvendo triang. sphæric. rectang., qvod determinatur per $d\Sigma$ et $(\alpha' - 90^\circ)$ obtinetur variatio arcus $d\alpha$; α hac correctum jam proxime verum est, eodemqve cursu semel repetito, certe verum erit.

40. Datis $\bar{\phi}$ ω Σ

Ex $\bar{\phi}$ ω obtinetur ϕ juxta form. 6; unde casus hic ad 39 reducitur.

41. Datis $\bar{\phi}$ ϕ Σ

Ex $\bar{\phi}$ ϕ obtinetur ω juxta form. 5; unde casus hic ad 39 reducitur.

42. Datis $\bar{\phi}$ Σ $\bar{\Sigma}$

Ex $\bar{\phi}$ $\bar{\Sigma}$ obtinetur ϕ juxta form. 9; unde casus hic ad 41 reducitur.

43. Datis ϕ Σ $\bar{\Sigma}$

Ex ϕ $\bar{\Sigma}$ obtinetur $\bar{\phi}$ juxta form. 10; unde casus hic ad 41 reducitur.

44. Datis ω Σ $\bar{\Sigma}$

Supponatur ϕ ex ω $\bar{\Sigma}$ per trig. sphæric. deductum; ex ϕ $\bar{\Sigma}$ obtinetur $\bar{\phi}$ juxta form. 10; ω ex $\bar{\phi}$ ϕ juxta form. 5; notatur $d\omega$, differentia inter ω datum et

comp.; differentiando ω ratione ϕ obtinetur variatio $d\phi$; ϕ hac correctum meliorem constituet hypothesin, qva idem cursus repeti potest. Vero ϕ invento, casus hic ad 39 reducitur.

45. Datis ϕ' ω $\bar{\Sigma}$

Qværatur ϕ via in casu 44 præscripta; qvo invento casus hic ad 1 reducitur.

46. Datis ω Σ $\bar{\Sigma}$

Qværatur ϕ via in casu 44 præscripta; qvo invento casus hic ad 12 reducitur.

47. Datis ω α' $\bar{\Sigma}$

Qværatur ϕ via in casu 44 præscripta; qvo invento casus hic ad 14 reducitur.

48. Datis ω $\bar{\Sigma}$ ψ

Qværatur ϕ via in casu 44 præscripta; qvo invento casus hic ad 28 reducitur.

9.

49. Datis ω α' Σ

Supponatur ϕ' ex ω α' Σ per trig. sphæric. deductum; ex $\phi' \alpha' \omega$ obtinetur ϕ juxta form. 2. a; Σ ex $\phi' \alpha' \phi$ juxta form. 3. a; notatur $d\Sigma$, differentia inter Σ datum et comput.; differentiando Σ ratione ϕ' obtinetur variatio $d\phi'$; ϕ' hac correctum meliorem constituet hypothesin, qva idem cursus repeti potest.

50. Datis $\bar{\phi}$ α' Σ

Supponatur ϕ' ex $\bar{\phi}$ α' Σ per trig. sphæric. deductum; ex $\phi' \alpha' \Sigma$ obtinetur ϕ juxta form. 4. a; ω ex $\phi' \alpha' \phi$ juxta form. 1. a; $\bar{\phi}$ ex $\phi \omega$ juxta form. 7; notatur $d\bar{\phi}$ differentia inter $\bar{\phi}$ datum et comput.; differentiando $\bar{\phi}$ ratione ϕ' obtinetur variatio $d\phi'$, qva si corrigatur ϕ' , probatior suppeditabitur hypothesis cursus ejusdem repetendi.

51. Datis ϕ' α' $\bar{\Sigma}$

Supponatur ϕ ex $\phi' \alpha' \bar{\Sigma}$ per trig. sphæric. deductum; obtinetur ω ex $\phi' \alpha' \phi$ per form. 1. a; $\bar{\phi}$ ex $\phi \omega$ per form. 7; $\bar{\Sigma}$ ex $\bar{\phi} \phi$ per form. 8; notatur $d\bar{\Sigma}$, differentia inter $\bar{\Sigma}$ datum et comput.; differentiando $\bar{\Sigma}$ ratione ϕ , obtinetur variatio $d\phi$, qva ad ϕ corrigendum adhibita, correctiori inde orto ϕ calculus idem repetendus adstruetur.

52. Datis $\phi' \Sigma \bar{\Sigma}$
 Supponatur α' ex $\phi' \Sigma \bar{\Sigma}$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi' \alpha' \Sigma$ per form. 4. a; ω ex $\phi' \alpha' \phi$ per form. 1. a; $\bar{\phi}$ per $\phi \omega$ ex form. 7; $\bar{\Sigma}$ ex $\bar{\phi} \phi$ per form. 8; notatur $d\bar{\Sigma}$, differentia inter $\bar{\Sigma}$ datum et comput.; differentiando $\bar{\Sigma}$ ratione α' prodit variatio $d\alpha'$; α' hac correctum novam consti-
tuet hypothesisin.
53. Datis $\phi \alpha' \bar{\Sigma}$
 Supponatur ϕ' ex $\phi \alpha' \bar{\Sigma}$ per trig. sphæric. deductum; obtinetur ω ex $\phi' \alpha' \phi$ juxta form. 1. a; $\bar{\phi}$ ex $\phi \omega$ juxta form. 7; $\bar{\Sigma}$ ex $\phi' \bar{\phi}$ juxta form. 11; obser-
vatur $d\bar{\Sigma}$, differentia inter $\bar{\Sigma}$ datum et comput.; differentiando $\bar{\Sigma}$ ratione ϕ' obtinetur variatio $d\phi'$, qva ϕ' correctum novæ adoptabitur hypothesis. Vero
 ϕ' invento casus hic ad 19 reducitur.
54. Datis $\phi \Sigma \bar{\Sigma}$
 Supponatur α ex $\phi \Sigma \bar{\Sigma}$ per trig. sphæric. deductum; ex $\phi \Sigma \alpha$ obtinetur ϕ' juxta form. 4. b; ω ex $\phi \alpha \phi'$ juxta form. 1. b; $\bar{\phi}$ ex $\phi \omega$ juxta form. 7; $\bar{\Sigma}$ ex $\phi' \bar{\phi}$ juxta form. 11; notatur $d\bar{\Sigma}$, differentia inter $\bar{\Sigma}$ datum et comput.; dif-
ferentiando $\bar{\Sigma}$ ratione α obtinetur variatio $d\alpha$. Nova deinde hypothesis va-
lore α per istam correctionem emendato superstruetur. Invento vero α ob-
tinetur α' juxta form. 14.
55. Datis $\omega \alpha' \bar{\Sigma}$
 Supponatur ϕ' ex $\omega \alpha' \bar{\Sigma}$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi' \omega \alpha'$ juxta form. 2. a; $\bar{\phi}$ ex $\phi \omega$ per form. 7; $\bar{\Sigma}$ ex $\bar{\phi} \phi'$ per form. 11; notatur $d\bar{\Sigma}$, differentia inter $\bar{\Sigma}$ datum et comput.; differentiando $\bar{\Sigma}$ ratione ϕ' obti-
netur variatio $d\phi'$; qva si corrigatur ϕ' , melior exhibetur hypothesis, qva
idem cursus repeti potest. Invento vero ϕ' casus hic ad 20 reducitur.
56. Datis $\omega \Sigma \bar{\Sigma}$
 Supponatur ϕ' ex $\omega \Sigma \bar{\Sigma}$ per trig. sphæric. deductum; ex $\phi' \Sigma$ obtinetur ϕ per form. 12; ϕ ex $\bar{\phi} \omega$ per form. 6; α' ex $\phi \omega \phi'$ per form. 2. c; Σ ex $\phi' \alpha' \phi$ per form. 3. a; observatur $d\bar{\Sigma}$, differentia inter $\bar{\Sigma}$ datum at comput.; diffe-

rentiendo Σ ratione ϕ' obtinetur variatio $d\phi'$; qva si corrigitur ϕ' , in ϕ' correcto melior sistetur hypothesis, qva calculus prorsus similis superstrui debet.

57. Datis $\phi' \phi \psi$

Supponatur α' ex $\phi' \phi \psi$ per trig. sphæric. deductum; obtinentur $\omega \alpha$ ex $\phi' \alpha' \phi$ per formm. 1, a, 14; $\bar{\phi}$ ex $\phi \omega$ per form. 7; $\bar{\alpha}$ ex $\phi \bar{\phi}$ per form. 15; ψ ex $\alpha \bar{\alpha}$ per form. 16; notatur $d\psi$, differentia inter ψ datum et comput.; differentiando ψ ratione α' obtinetur variatio $d\alpha'$; α' hac correctum meliorum suppeditabit hypothesisin, qva idem cursus repeti potest. Tandem ex $\phi' \phi \alpha'$ obtinetur Σ juxta form. 3. a.

58. Datis $\phi' \omega \psi$

Supponatur α' ex $\phi' \omega \psi$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi' \alpha' \omega$ juxta form. 2. a; α ex $\phi' \alpha' \phi$ per form. 14; $\bar{\phi}$ ex $\phi \omega$ per form. 7; $\bar{\alpha}$ ex $\phi \bar{\phi}$ per form. 15; ψ ex $\alpha \bar{\alpha}$ per form. 16; notatur $d\psi$ differentia inter ψ datum et comput.; differentiando ψ ratione α' obtinetur variatio $d\alpha'$; qvæ si in suppositum α' adhibetur, longe meliorem nanciscemur hypothesisin, qva idem calculus repeti potest. Tandem habetur Σ ex $\phi' \alpha' \phi$ juxta form. 3. a.

59. Datis $\phi' \alpha' \psi$

Supponatur ω ex $\phi' \alpha' \psi$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi' \alpha' \omega$ juxta form. 2. a; α ex $\phi' \alpha' \phi$ per form. 14; $\bar{\phi}$ ex $\phi \omega$ per form. 7; $\bar{\alpha}$ ex $\phi \bar{\phi}$ per form. 15; ψ ex $\alpha \bar{\alpha}$ per form. 16; notatur $d\psi$, differentia inter ψ datum et comput.; differentiando ψ ratione ω obtinetur variatio $d\omega$, qva si corrigitur ω , melior oritur suppositio, cui calculus repetendus adstrui potest. Tandem obtinetur Σ ex $\phi' \alpha' \phi$ per form. 3. a.

60. Datis $\phi' \Sigma \psi$

Supponatur α' ex $\phi' \Sigma \psi$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi' \alpha' \Sigma$ per form. 4. a; $\omega \alpha$ ex $\phi' \alpha' \phi$ et sic porro ut in casu 57.

61. Datis $\phi' \bar{\Sigma} \psi$

Supponatur ϕ ex $\phi' \bar{\Sigma} \psi$ per trig. sphæric. deductum; obtinetur $\bar{\phi}$ ex $\phi \bar{\Sigma}$ per form. 10; $\omega \bar{\alpha}$ per formm. 5, 15; $\bar{\alpha}$ ex $\psi \bar{\alpha}$ per form. 16; ϕ' ex $\phi \omega \alpha$ per form. 2. b; notatur $d\phi'$, differentia inter ϕ' datum et comput.; differentiando ϕ' ratione ϕ obtinetur variatio $d\phi$; qva si corrigitur ϕ , melior

prohibet hypothesis calculi eadem via repetendi. Tandem habetur Σ ex $\phi' \alpha' \psi$ per form. 3. a.

62. Datis $\phi \alpha' \psi$

Supponatur ω ex $\phi \alpha' \psi$ per trig. sphæric. deductum; ex $\phi \omega$ obtinetur ϕ per form. 7; α ex $\phi \phi$ per form. 15; α ex $\psi \alpha$ per form. 16; ϕ' ex $\phi \omega \alpha$ per form. 2. b; α' ex $\phi' \alpha \phi$ per form. 14; notatur $d\alpha'$, differentia inter α' datum et comput.; differentiando α' ratione ω obtinetur variatio $d\omega$; per quam si corrigatur ω , melior obtinetur valor, quo idem cursus repeti potest. Unde casus hic ad 14 reducitur.

63. Datis $\phi \alpha' \psi$

Supponatur ω ex $\phi \alpha' \psi$ per trig. sphæric. deductum; ex $\phi \omega$ obtinetur ϕ juxta form. 6; α ex $\phi \phi$ et sic porro ut in casu 62.

64. Datis $\phi \Sigma \psi$

Supponatur ω ex $\phi \Sigma \psi$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi \omega$ per form. 7; α ex $\phi \phi$ per form. 15; α ex $\psi \alpha$ per form. 16; ϕ' ex $\phi \omega \alpha$ per form. 2. b; Σ ex $\phi \alpha \phi'$ per form. 3. b; notatur $d\Sigma$, differentia inter Σ datum et comput.; differentiando Σ ratione ω obtinetur variatio $d\omega$; ω hac correctum meliorem constituet hypothesis, qua idem cursus repeti potest.

65. Datis $\phi \Sigma \psi$

Supponatur ω ex $\phi \Sigma \psi$ per trig. sphæric. deductum; ex $\phi \omega$ obtinetur ϕ juxta form. 6; α ex $\phi \phi$ et sic porro ut in casu 64.

66. Datis $\omega \alpha' \psi$

Supponatur ϕ' ex $\omega \alpha' \psi$ per trig. sphæric. deductum; ex $\phi' \alpha' \omega$ obtinetur ϕ juxta form. 2. a; ϕ ex $\phi \omega$ juxta form. 7; α ex $\phi \phi$ per form. 15; α ex $\phi' \alpha' \phi$ per form. 14; ψ ex $\alpha \alpha$ per form. 16; notatur $d\psi$ differentia inter ψ datum et comput.; differentiando ψ ratione ϕ' obtinetur variatio $d\phi'$; qua si emendetur ϕ' , potiorem accipiemus hypothesis, qua idem cursus repeti potest. Tandem Σ ex $\phi' \alpha' \phi$ per form. 3. a.

67. Datis $\omega \Sigma \psi$

Supponatur ϕ ex $\omega \Sigma \psi$ per trig. sphæric. deductum; obtinetur ϕ ex $\phi \omega$ per form. 7; α ex $\phi \phi$ per form. 15; α ex $\psi \alpha$ per form. 16; ϕ' ex $\phi \omega \alpha$ per form. 2. b; Σ ex $\phi \alpha \phi'$ per form. 3. b; notatur $d\Sigma$ differentia inter Σ datum

et comput.; differentiando Σ ratione ϕ obtinetur variatio $d\phi$; qva si corrigitur ϕ , meliori potiemur conditione, ad qvam idem calculus repeti potest.

68. Datis $\phi \stackrel{1}{\Sigma} \psi$

Supponatur $\bar{\phi}$ ex $\phi \stackrel{1}{\Sigma} \psi$ per trig. sphæric. deductum; ex $\bar{\phi} \bar{\phi}$ obtinetur ω per form. 5; α ex $\bar{\phi} \bar{\phi}$ per form. 15; α ex $\psi \alpha$ per form. 16; ϕ' ex $\phi \omega \alpha$ per form. 2.b; $\bar{\Sigma}$ ex $\phi' \bar{\phi}$ per form. 11; notatur $d\bar{\Sigma}$, differentia inter $\bar{\Sigma}$ datum et comput.; differentiando $\bar{\Sigma}$ ratione $\bar{\phi}$ obtinetur variatio $d\bar{\phi}$, qva si corrigitur $\bar{\phi}$, novam obtinebimus hypothesin, qva idem cursus repeti potest. $\bar{\phi}$ invento casus hic ad 9 reducitur.

69. Datis $\omega \stackrel{1}{\Sigma} \psi$

Supponatur $\bar{\phi}$ ex $\omega \stackrel{1}{\Sigma} \psi$ per trig. sphæric. deductum; ex $\bar{\phi} \omega$ obtinetur ϕ juxta form. 6; α ex $\bar{\phi} \bar{\phi}$ et sic porro ut in casu 68.

IO.

Solvendi supersunt casus decem, nimirum

$$\begin{array}{ll} \alpha' \Sigma \bar{\Sigma}; & \alpha' \Sigma \stackrel{1}{\Sigma}; \\ \alpha' \Sigma \psi; & \alpha' \bar{\Sigma} \stackrel{1}{\Sigma}; \\ \alpha' \bar{\Sigma} \psi; & \alpha' \bar{\Sigma} \bar{\Sigma}; \\ \Sigma \bar{\Sigma} \stackrel{1}{\Sigma}; & \Sigma \bar{\Sigma} \psi; \\ \Sigma \bar{\Sigma} \bar{\Sigma}; & \Sigma \bar{\Sigma} \bar{\Sigma} \psi; \\ \Sigma \bar{\Sigma} \psi; & \Sigma \bar{\Sigma} \bar{\Sigma} \psi; \\ \Sigma \bar{\Sigma} \bar{\Sigma} \psi; & \Sigma \bar{\Sigma} \bar{\Sigma} \bar{\Sigma}; \\ \Sigma \bar{\Sigma} \bar{\Sigma} \bar{\Sigma} \psi; & \Sigma \bar{\Sigma} \bar{\Sigma} \bar{\Sigma} \bar{\Sigma}. \end{array}$$

Intercedit inter hos jam dictos casus et illos supra indirecte solutos memorabilis differentia hæc, qvod via ad valores approximatos ex natura ipsius problematis petendos, qvæ in illis patuit, in his interclusa videtur. Qvamobrem, nisi approximati isti valores cæcis quasi tentaminibus sæpisime repetitis eruti fuerint, solutionis expertes manebunt decem hi casus, donec datis novæ accesferint conditiones; qvæ utiqve praxi haud deerunt, qvas autem hic nunc evolvere nec succedit nec conveniet. Mentione dignum est, posterioribus duobus horum casuum aliud plane novum inhærere incommodum, qvod hi nempe, ut ad casus 1-28 reduci posint, non unum tantum, ut cæteri indirecte solyendi casus, sed bina desiderent elementa.

(ad hanc) sequitur latitudineos et coindenitatem **III.** Situosa istice ut obnoxio exponitur

Qvo perspecto formularum **I**-**16** usu, jam provocati videmur, ut, diligenter subducta ratione, qvanta sit superiorum excentricitatis potestatum hactenus neglectarum vis, explorare studeamus.

Ut supra orsi a trita ellipsis, æquatione

$$y^2 = (1 - ee)(a^2 - x^2)$$

iterum conseqvimur

$$\text{tang. } \phi = -\frac{dx}{dy} = \frac{1}{x} \cdot \frac{\sqrt{a^2 - x^2}}{\sqrt{1 - ee}}$$

et præterea

$$\sin. \phi^2 = \frac{a^2 - \rho^2}{a^2 - e^2 \rho^2}$$

$$P = \frac{(a^2 - e^2 \rho^2)^{\frac{3}{2}}}{a^2 \sqrt{1 - e^2}}$$

$$d\phi = \frac{-a^2 \sqrt{1 - ee} \cdot d\rho}{\sqrt{a^2 - \rho^2} (a^2 - e^2 \rho^2)}$$

qvorum duobus posterioribus in æquatione **G** et **H** substitutis
prodit

$$d\omega = \frac{-\rho' \sin. \alpha' d\rho}{\xi \sqrt{\rho^2 - \rho'^2} \sin. \alpha'^2} \sqrt{\frac{a^2 - e^2 \rho^2}{a^2 - \rho^2}}$$

$$d\Sigma = \frac{-\rho d\rho}{\sqrt{\rho^2 - \rho'^2} \sin. \alpha'^2} \sqrt{\frac{a^2 - e^2 \rho^2}{a^2 - \rho^2}}$$

sive, si ponantur

$$q^2 = \frac{a^2 - \rho^2}{\rho^2 - \rho'^2 \sin. \alpha'^2} = \frac{a^2 - \rho^2}{\rho^2 - \beta^2}$$

$$\xi^2 = \frac{a^2 + \vartheta^2 q^2}{1 + q^2}$$

$$\xi d\xi = -q dq \frac{(a^2 - \beta^2)}{(1 + q^2)^2}$$

$$\xi^2 - \vartheta^2 = \frac{a^2 - \beta^2}{1 + q^2}$$

$$\text{tum } d\omega = \frac{\vartheta dq}{a^2 + \vartheta^2 q^2} \sqrt{a^2 - e^2 \frac{(a^2 + \vartheta^2 q^2)}{1 + q^2}}$$

$$d\Sigma = \frac{dq}{1 + q^2} \sqrt{a^2 - e^2 \frac{(a^2 + \vartheta^2 q^2)}{1 + q^2}}$$

(D)

utrumque evolvendo in series secundum potestates excentricitatis progredientes, prodit, neglectis sexti ordinis altioribusque potestatibus.

$$d\omega = \frac{\vartheta dq}{a^2 + \vartheta^2 q^2} \left\{ a - \frac{1}{2} e^2 \frac{a^2 + \vartheta^2 q^2}{a(1+q^2)} - \frac{1}{8} e^4 \frac{(a^2 + \vartheta^2 q^2)^2}{a^3(1+q^2)^2} \right\}$$

$$d\Sigma = -\frac{dq}{1+q^2} \left\{ a - \frac{1}{2} e^2 \frac{a^2 + \vartheta^2 q^2}{a(1+q^2)} - \frac{1}{8} e^4 \frac{(a^2 + \vartheta^2 q^2)^2}{a^3(1+q^2)^2} \right\}$$

sive

$$d\omega = \frac{a \vartheta dq}{a^2 + \vartheta^2 q^2} - \frac{1}{2} e^2 \frac{\vartheta}{a} \frac{dq}{1+q^2} - \frac{1}{8} e^4 \frac{\vartheta}{a^3} \frac{(a^2 + \vartheta^2 q^2)}{(1+q^2)^2} dq$$

$$d\Sigma = \frac{a dq}{1+q^2} - \frac{1}{2} e^2 \frac{(a^2 + \vartheta^2 q^2)}{a} \frac{dq}{(1+q^2)^2} - \frac{1}{8} e^4 \frac{(a^2 + \vartheta^2 q^2)^2}{a^3} \frac{dq}{(1+q^2)^3}$$

Membrum ultimum ipsius $d\omega$ est

$$= \frac{e^4}{16} \frac{\vartheta}{a^3} \left\{ d \left(\frac{a^2 + \vartheta^2 q^2}{q(1+q^2)} \right) + a^2 \frac{dq}{q^2} - (a^2 + \vartheta^2) \frac{dq}{1+q^2} \right\}$$

Secundum ipsius $d\Sigma$ est

$$= -\frac{e^2}{4} \left\{ -d \left(\frac{a^2 + \vartheta^2 q^2}{q(1+q^2)} \right) - a^2 \frac{dq}{q^2} + (a^2 + \vartheta^2) \frac{dq}{1+q^2} \right\}$$

Ultimum ipsius $d\Sigma$ est

$$= \frac{e^4}{64} \frac{1}{a^3} \left\{ 2d \left(\frac{(a^2 + \vartheta^2 q^2)^2}{q(1+q^2)^2} \right) + (a^2 + \vartheta^2) d \left(\frac{a^2 + \vartheta^2 q^2}{q(1+q^2)} \right) + 3a^2(a^2 + \vartheta^2) \frac{dq}{q^2} - (3(a^2 + \vartheta^2)^2 - 4a^2\vartheta^2) \frac{dq}{1+q^2} \right\}$$

Unde sequitur

$$\omega = \text{arc} \left\{ \tan \left(\frac{\vartheta}{a} \cdot q \right) \right\} - \frac{e^2 \vartheta}{2 \cdot a} \text{arc}(\tan = q) + \frac{e^4 \vartheta}{16 a^3} \left\{ \frac{a^2 + \vartheta^2 q^2}{q(1+q^2)} - \frac{a^2}{q} - (a^2 + \vartheta^2) \text{arc.}(\tan = q) \right\}$$

$$\Sigma = a + \text{arc.}(\tan = q) - \frac{e^2}{4} \cdot \frac{1}{a} \left\{ -\frac{a^2 + \vartheta^2 q^2}{q(1+q^2)} + \frac{a^2}{q} + (a^2 + \vartheta^2) \text{arc.}(\tan = q) \right\}$$

$$+ \frac{e^4}{64} \cdot \frac{1}{a^3} \left\{ 2 \frac{(a^2 + \vartheta^2 q^2)^2}{q(1+q^2)^2} + (a^2 + \vartheta^2) \frac{(a^2 + \vartheta^2 q^2)}{q(1+q^2)} - 3a^2 \frac{(a^2 + \vartheta^2)}{q} - (3(a^2 + \vartheta^2)^2 - 4a^2\vartheta^2) \text{arc.}(\tan = q) \right\}$$

sive, membris contractis

$$\omega = \text{arc.} \left(\tan \left(\frac{\vartheta}{a} q \right) \right) - \frac{\vartheta}{a} \text{arc.}(\tan = q) \left\{ \frac{1}{2} e^2 + \frac{e^4 (a^2 + \vartheta^2)}{16 a^2} \right\} - \frac{e^4 \vartheta}{16 a^3} \frac{(a^2 - \vartheta^2)}{1+q^2} q + \text{const.}$$

$$\Sigma = a \text{arc.}(\tan = q) \left\{ 1 - \frac{e^2 a^2 + \vartheta^2}{4 a^2} - \frac{e^4}{64} \left(\frac{5(a^2 + \vartheta^2)^2 - 4a^2\vartheta^2}{a^4} \right) \right\} - \frac{e^2 (a^2 - \vartheta^2)}{4 a} \frac{q}{1+q^2}$$

$$- \frac{e^4}{64} \cdot \frac{1}{a^3} \left\{ 2a^2(a^2 - 2\vartheta^2) \frac{q}{(1+q^2)^2} - 2\vartheta^4 \frac{q^3}{(1+q^2)^2} + (3(a^4 - \vartheta^4) + 2a^2\vartheta^2) \frac{q}{1+q^2} \right\} + \text{const.}$$

Utroque integrali inde $\phi = \phi'$ sive $q = q'$ sumto

$$\begin{aligned} a &= \text{arc.} \left(\tan g. \frac{\vartheta}{a}, q \right) - \text{arc.} \left(\tan g. \frac{\vartheta}{a}, q' \right) - \frac{\vartheta}{a} \left\{ \text{arc.} (\tan g. q) - \text{arc.} (\tan g. q') \right\} \left\{ \frac{e^2}{2} + \frac{e^4 (a^2 + \vartheta^2)}{16 a^2} \right\} \\ &\quad - \frac{e^4}{16} \frac{\vartheta}{a^3} (a^2 - \vartheta^2) \left\{ \frac{q}{1+q^2} - \frac{q'}{1+q'^2} \right\} \\ \Sigma &= a \left\{ \text{arc.} (\tan g. q) - \text{arc.} (\tan g. q') \right\} \cdot \left\{ 1 - \frac{e^2 (a^2 + \vartheta^2)}{4 a^2} - \frac{e^4}{64} \left(\frac{3(a^2 + \vartheta^2)^2 - 4a^2 \vartheta^2}{a^4} \right) \right\} \\ &\quad - \frac{e^2 (a^2 - \vartheta^2)}{4 a} \left\{ \frac{q}{1+q^2} - \frac{q'}{1+q'^2} \right\} - \frac{e^4}{64} \cdot \frac{1}{a^3} \left\{ 2 a^2 (a^2 - 2 \vartheta^2) \left(\frac{q}{(1+q^2)^2} - \frac{q'}{(1+q'^2)^2} \right) \right. \\ &\quad \left. - 2 \vartheta^4 \left(\frac{q^3}{(1+q^2)^2} - \frac{q'^3}{(1+q'^2)^2} \right) + (3(a^4 - \vartheta^4) + 2a^2 \vartheta^2) \left(\frac{q}{1+q^2} - \frac{q'}{1+q'^2} \right) \right\} \end{aligned}$$

Statuendo

$$\cos. g = \frac{\vartheta}{a}$$

$$\tan g. n = q$$

$$\tan g. m = \cos. g \tan g. n$$

conseqvimus

$$\begin{aligned} \omega &= m - m' - \cos. g (n - n') \left\{ \frac{e^2}{2} + \frac{e^4}{16} (1 + \cos. g^2) \right\} - \frac{e^4}{16} \cos. g \sin. g^2 \sin. (n - n') \cos. (n + n') \\ \Sigma &= a (n - n') \left\{ 1 - \frac{e^2}{4} (1 + \cos. g^2) - \frac{e^4}{64} (3(1 + \cos. g^2)^2 - 4 \cos. g^2) \right\} \\ &\quad - a \frac{e^2}{4} \sin. g^2 \sin. (n - n') \cos. (n + n') \left\{ 1 + \frac{e^2}{16} (5 + 3 \cos. g^2) \right\} \\ &\quad + a \frac{e^4}{32} \sin. g^4 \left\{ \sin. n^3 \cos. n - \sin. n'^3 \cos. n' \right\} \end{aligned}$$

Tandem venia supra data, potestates excentricitatis negligendi, variis modis utentes,

$$\begin{aligned} \text{introducto} &\quad \tan g. x = \frac{\sin. n'^3 \cos. n'}{\sin. n^2 \cos. n^2} \\ \text{impetramus} &\quad \omega = m - m' - \frac{\frac{1}{2} e^2}{\sqrt{1-e^2}} \cos. g (n - n') \left\{ 1 + \frac{e^2}{8} \cos. g^2 \right\} - \frac{e^4}{16} \cos. g \sin. g^2 \sin. (n - n') \cos. (n + n') \end{aligned}$$

$$\begin{aligned} \Sigma &= a \sqrt{1-e^2} (n - n') + \frac{\frac{1}{4} e^2 a}{\sqrt{1-e^2}} \sin. g^2 \left\{ 1 - \frac{3}{16} e^2 \sin. g^2 \right\}, \left\{ n - n' - \sin. (n - n') \cos. (n + n') \right\} \\ &\quad + \frac{e^4}{32} a \sqrt{1-e^2} \frac{\cos. n}{\cos. x} \sin. g^4 \sin. n^2 \sin. (n - x) \end{aligned}$$

Collectis æquationibus conditionalibus

$$\tan. f' = \sqrt{1-ee} \tan. \phi'$$

$$\cos. g = \sin. \alpha' \cos. f' \quad \text{ad calc. comprob.} \quad \sin. g = \frac{\sin. f'}{\sin. n'}$$

$$\tan. m' = \tan. \alpha' \sin. f' \quad \sin. m' = \tan. f' \cot. g$$

$$\tan. n' = \frac{\tan. f'}{\cos. \alpha'} \quad \cos. n' = \frac{\cos. \alpha' \cos. f'}{\sin. g}$$

$$\tan. f = \sqrt{1-ee} \tan. \phi \quad \tan. m = \cos. g \tan. n$$

$$\sin. m = \tan. f \cot. g \quad \tan. (45^\circ - \frac{n}{2}) = \sqrt{\tan. (\frac{g-f}{2}) \cot. (\frac{g+f}{2})}$$

$$\sin. n = \frac{\sin. f}{\sin. g} \quad \sin. d = \sin. \alpha' \frac{\sin. l}{\cos. f}$$

$$n - n' = l \quad ; \quad d = m - m'$$

prodit

$$\omega = d - \frac{\frac{1}{2}ee}{\sqrt{1-ee}} \cos. g \cdot l \left(1 + \frac{e^2}{8} \cos. g^2 \right) - \frac{e^4}{16} \frac{\cos. g}{\sin. n'} \sin. g^2 \sin. l \cos. (n+n')$$

formula, qva ω ex $\phi' \alpha' \phi$ obtinetur. (i. a)

Surrogando α in locum ipsius α' et ϕ' cum ϕ permutando, eadem ratione ω ex $\phi' \alpha \phi$ eruitur. (i. b)

Jam per theorema, qvòd nomen Lagrange gerit, hæc series converti potest, etenim retentis æquationum conditionis antecedentium quatuor prioribus et introducendo

$$\text{tang. } N = \frac{\tan. (\omega \pm m')}{\cos. g} \quad \text{nanciscemur}$$

$$\begin{aligned} d &= \omega + \frac{\frac{1}{2}ee}{\sqrt{1-ee}} \cos. g (N-n') \left\{ 1 + \frac{e^2 - \cos. N^2}{2 \cos. (\omega \pm m')^2} \left(1 + \frac{\cos. g^2 \cos. (\omega \pm m')^2}{4 \cos. N^2} \right) \right\} \\ &\quad + \frac{e^4 \cos. g}{16 \sin. n'} \sin. g^2 \sin. (N-n') \cos. (N+n') \end{aligned} \quad (2)$$

unde, propter æquationes

$$m = d + m' \quad \text{m = d + m'}$$

$$\tan. f = \sin. m \tan. g$$

$$\tan. \phi = \frac{\tan. f}{\sqrt{1-ee}}$$

ϕ ex $\phi' \alpha' \omega$ datis conseqvimus (2. a)

Si α ipsi α' sufficiatur et ϕ' cum ϕ permuteatur, similiter ϕ' ex ϕ α ω erui potest (2, b)

Perscrutantes deinde valorem α' , datis $\phi' \phi \omega$, juvabit illa series (2) quum enim

$$\text{introducto} \quad \tan g. k = \frac{\tan g. f}{\cos. d}$$

$$\text{facile demonstretur} \quad \cot \alpha' = \cot d \frac{\sin(k-f')}{\cos k} = \frac{\tan f \sin(k-f')}{\sin k \sin d}$$

jam supponentes $d = \omega$, per æquationem allatam valorem α' approximatum eruere possumus; qvo invento si computentur æquationes conditionales seriei (2) una cum ipsa serie, correctum obtinebitur d . Correcto d in locum suppositi substituto, meliorem nanciscemur hypothesis, qva idem cursus repeti potest, donec et d et α' exactum prodibit (2. c)

Pariter ϕ' ex $\alpha' \phi \omega$ vi ejusdem seriei (2) eruetur, etenim

insinuantes tang. $i \equiv \cos d \cot f$

facile probabimus $\cos(f' + i) = \tan d \cot \alpha' \sin i = \sin d \cot \alpha' \cos i \cot f'$ æquationem, qva, $d = \omega$ supposito, $f' + i$ adeoqve f' approximate inveniri possit. Proinde approximato f' æquationes conditionales seriei (2) et ipsam seriem computantes, correctum d impetrabimus, atqve hoc ipsum in locum suppositi substituentes, tandem, cursu eodem repetito, in verum f' et ϕ' incidemus (2. d)

Præmissis æquationibus

tang, $f' = \sqrt{1-ee} \tan \varphi'$ Ad calc, comprob.

$$\cos, g = \sin, \alpha' \cos, f' \quad \text{and} \quad \sin, g = \frac{\sin, f'}{\sin, n'}$$

$$\tan \theta = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \quad \text{and} \quad \sin \theta = \sqrt{\frac{2 \sin^2 \alpha}{1 + \cos \alpha}}$$

$$\sin. n = \frac{\sin f}{\sin g} \quad \text{tang.}(45^\circ - \frac{n}{2}) = \sqrt{\tan(\frac{g-f}{2}) \cot(\frac{g+f}{2})}$$

$$l = n - n'$$

$$\text{tang. } x = \frac{\sin n^\circ}{\cos n^\circ} = \frac{\sin n^\circ}{z \cos n^\circ} = \frac{\sin n^\circ}{z}$$

habetur

$$\sigma = l + \frac{e^2 \sin. g^2}{4(1-e)} \cdot l \left\{ 1 - \frac{\sin. l \cos. (n+\pi n')}{\sin. 1'' l} \right\} \left\{ 1 - \frac{3}{16} e^2 \sin. g^2 \right\} + \frac{e^4 \sin. g^4 \cos. n}{32 \sin. 1'' \cos. x} \sin. n^2 \sin. (n-x)$$

$$\Sigma \equiv a\sqrt{1-ee} \sigma \sin i^{\circ}$$

formula, qva Σ ex $\varphi' \alpha' \varphi$ obtinetur (3.2)

Ponendo α pro α' et β' cum β permutando, Σ ex α' & β pariter conseqimur (3. b)

Converti potest illa series, quum enim antecedentium aeqvationum conditionum tres priores retineantur et introducatur

$$(4) \text{ aeqv. } \tan. X = \frac{\sin. n^3 \cos. n'}{\sin. (\sigma + n')^2 \cos. (\sigma + n')}$$

prodit

$$\begin{aligned} l &= \sigma - \frac{e^2 \sin. g^2}{4 \cdot 1 - ee} \sigma \left\{ 1 - \frac{\sin. \sigma \cos. (\sigma + 2n')}{\sin. 1'' \sigma} \right\} \left\{ 1 - \frac{3}{16} e^2 \sin. g^2 \left(1 + \frac{8}{3} \sin. (\sigma + n')^2 \right) \right\} \\ &\quad - \frac{e^4 \sin. g^4 \cos. (\sigma + n')}{32 \sin. 1'' \cos. X} \sin. (\sigma + n')^2 \sin. (\sigma + n' - X) \dots \end{aligned} \quad (4)$$

unde propter aeqvationes

$$n = l + n'$$

$$\sin. f = \sin. n \sin. g$$

$$\tan. \phi = \frac{\tan. f}{\sqrt{1 - ee}}$$

ϕ ex $\phi' \alpha' \Sigma$ datis indagari potest (4. a)

Si α sufficiatur ipsi α' et ϕ' cum ϕ permutetur, similiter ϕ' ex $\phi \alpha \Sigma$ eruetur . (4. b)

Ex ipsa hac serie (4) peti potest praesidium ad α' ex $\phi' \phi \Sigma$ eruendum; nam siquidem

introducto $\tan. z = \frac{\cos. l \cos. f'}{\sin. f}$

$$\text{manifesto est } \cos. \alpha' = \frac{\sin. (f-z)}{\cos. z \cos. f' \sin. l} = \frac{\tan. f' \cot. l}{\sin. z \cos. f'} \sin. (f-z)$$

jam supponentes $l = \sigma$, valorem approximatim ipsius α' ex aeqvatione eliciemus, qvo aeqvationes conditionales seriei (4) et ipsam seriem computabimus, unde existet correctus valor l . Correcto l in locum suppositi surrogato, eundem calculum repetentes, tandem verum α' asseqvemur (4. c)

Ipsissima hæc series datis $\phi \Sigma \alpha'$ largietur valorem ϕ' ; quum nimirum

$$\text{inferendo } \tan. t = \tan. l \cos. \alpha'$$

$$\text{constet } \sin. (f' + t) = \frac{\cos. t}{\cos. l} \sin. f = \frac{\sin. t \sin. f}{\sin. l \cos. \alpha'}$$

supponetur $l = \sigma$, et per aeqvationem jam expositam eruetur valor approximatus $f' + t$, unde f' . Cujus valore invento, series (4) una cum aeqvationibus præmissis computabitur; hinc correctum sistetur l . Substituentes correctum l in locum suppositi, via eadem repetita ad ipsum correctum f' et ϕ' adducemur . . . (4. d)

13.

Præmissis æquationibus:

$$\tang. \tilde{f} = \sqrt{1-e^2} \tang. \tilde{\phi}$$

$$\tang. f = \sqrt{1-e^2} \tang. \phi$$

ad calc. confirm.

$$\cos. \tilde{d} = \tang. f \cot. \tilde{f}$$

$$\tang. d = \frac{\tang. \tilde{f}}{\cos. \tilde{f}}$$

$$\cos. \tilde{l} = \frac{\sin. f}{\sin. \tilde{f}}$$

$$\tang. \frac{i}{2} = \pm \sqrt{\tang. \left(\frac{f-\tilde{f}}{2}\right) \cot. \left(\frac{f+\tilde{f}}{2}\right)}$$

existit

$$\omega \equiv \tilde{d} = \frac{\frac{1}{2}ee}{\sqrt{1-e^2}} \cos. f. \tilde{l} \left(1 + \frac{e^2}{8} \cos. \tilde{f}^2 \right) + \frac{e^4}{32} \frac{\cos. \tilde{f}}{\sin. \tilde{l}} \sin. \tilde{f}^2 \sin. 2\tilde{l}$$

formula, qva ω ex ϕ $\tilde{\phi}$ invenitur (5)

Duplici modo converti potest hæc series

partim

$$(6) \text{ ponendo } \tang. \tilde{f} = \sqrt{1-e^2} \tang. \tilde{\phi}$$

$$\text{et introducendo } \tang. \tilde{L} = \tang. \omega \cos. \tilde{f}$$

ut inde profiscatur

$$\tilde{d} = \omega + \frac{\frac{1}{2}ee}{\sqrt{1-e^2}} \cos. \tilde{f} \tilde{L} \left\{ 1 + \frac{e^2}{8} \cos. \tilde{f}^2 \left(1 + 4 \frac{\cos. \tilde{L}^2}{\cos. \omega^2} \right) \right\} - \frac{e^4}{32} \frac{\cos. \tilde{f}}{\sin. \tilde{l}} \sin. \tilde{f}^2 \sin. 2\tilde{L}$$

et propter æquationes

$$\tang. f = \cos. \tilde{d} \tang. \tilde{f}$$

$$\tang. \phi = \frac{\tang. f}{\sqrt{1-e^2}}$$

valor ϕ ex datis $\tilde{\phi}$ ω concludatur (6)

partim

$$\text{ponendo } \tang. f = \sqrt{1-e^2} \tang. \phi$$

$$\tang. \tilde{F} = \frac{\tang. f}{\cos. \omega}$$

$$\tang. \tilde{L} = \tang. \omega \cos. \tilde{F}; \text{ sive } \cos. \tilde{L} = \frac{\sin. f}{\sin. \tilde{F}}$$

formetur series

$$\tilde{d} = \omega + \frac{\frac{1}{2}ee}{\sqrt{1-e^2}} \cos. \tilde{F} \tilde{L} \left\{ 1 + \frac{e^2}{16} \cos. \tilde{F}^2 \left(1 - 0.8 \frac{\sin. \tilde{l}. \tang. \tilde{L}}{\cos. \tilde{F}^2} \right) \right\} - \frac{e^4}{32} \frac{\cos. \tilde{F}}{\sin. \tilde{l}} \sin. \tilde{F}^2 \sin. 2\tilde{L}$$

et propter æquationes

$$\text{tang. } \bar{f} = \frac{\text{tang. } f}{\cos. \bar{d}}$$

$$\text{tang. } \tilde{\phi} = \frac{\text{tang. } \tilde{f}}{\sqrt{1-e^2}}$$

Præmisſis

$$\text{tang. } \tilde{f} = \sqrt{1-e_e} \text{ tang. } \tilde{\phi}$$

$$\text{tang. } f = \sqrt{1-e_e} \text{ tang. } \varphi$$

$$\cos. \bar{l} = \frac{\sin. f}{\sin. \bar{f}} ; \quad \text{; } \quad \text{tang. } \bar{l} = \pm \sqrt{\tan. \left(\frac{\bar{f}-f}{2} \right) \cot. \left(\frac{\bar{f}+f}{2} \right)}$$

prodit

$$\sigma = \bar{l} + \frac{\frac{e^4 ee}{1-ee}}{\sin. l''} \sin. f^2 \cdot \bar{l} \left\{ 1 + \frac{\sin. \tilde{l} \cos. \tilde{l}}{\sin. l'' \cdot \bar{l}} \right\} \left\{ 1 - \frac{3}{16} e^2 \sin. f^2 \right\} - \frac{e^4}{32} \frac{\sin. f^2}{\sin. l''} \sin. \tilde{l} \cos. \tilde{l} l^2$$

$$\Sigma \equiv a\sqrt{1-e^2} \sin. i'' \cdot \sigma$$

Præmissis

$$\sigma = \frac{\Sigma}{a V_{1-e} \sin i}$$

$$\tang. \tilde{f} = \sqrt{1-e^e} \tang. \tilde{\phi}$$

prodit

$$l = \sigma - \frac{\frac{1}{4}ee}{1-ee} \sin \sqrt{e^2 - e^2} \left\{ 1 + \frac{\sin \sigma \cos \sigma}{\sin 1'' \cdot \sigma} \right\} \left\{ 1 - \frac{3}{16} e^2 \sin \sqrt{e^2} \left(1 + \frac{8}{3} \cos \sigma^2 \right) \right\} + \frac{e^4 \sin \sqrt{e^2}}{32 \sin 1''} \sin \sigma \cos \sigma \cdot \sigma^3$$

unde, quum

$$\tan \varphi = \frac{\tan f}{\sqrt{1 - ee}}$$

ϕ ex datis $\bar{\phi}$ Σ obtinetur. (9)

Statuendo

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{M}{a\sqrt{1-e^x}} \sin x e^{-xt} dt$$

$$\text{tang. } f = \sqrt{1-e^2} \text{ tang. } \phi$$

$$\sin. \mathcal{F} = \frac{\sin. f}{\cos. f}$$

oritur

$$\bar{l} = \sigma - \frac{\frac{1}{4}ee}{1-ee} \sin \tilde{\mathcal{F}}^2 \cdot \sigma \left[1 + \frac{\sin \sigma \cos \sigma}{\sin \Gamma'' \sigma} \right] \left\{ 1 - \frac{1}{16}e^2 \sin \tilde{\mathcal{F}}^2 \left(1 + \frac{8}{15} \sin \Gamma'' \sigma \tanh \sigma \right) \right\} + \frac{e^4}{32} \cdot \frac{\sin \tilde{\mathcal{F}}^4}{\sin \Gamma''} \cdot \sin \sigma \cos \sigma^3$$

quare, quum

$$\sin \bar{f} = \frac{\sin f}{\cos l} \quad \text{sive } \tan(45^\circ - \frac{\bar{f}}{2}) = \sqrt{\tan\left(\frac{90^\circ - f - l}{2}\right) \tan\left(\frac{90^\circ - f + l}{2}\right)}$$

Adhibendo

$$\tang. f' = \sqrt{1-e} \tang. \phi'$$

$$\tang. \tilde{f} = \sqrt{1-e_e} \tang. \tilde{\phi}$$

$$l = \bar{f} - f'$$

$$\text{tang. } x = \frac{\sin f^1 \cos f^1}{\sin f^2 \cos f^2}$$

habetur

$$\frac{1}{\sigma} = l + \frac{1}{4} e^2 l \left\{ 1 - \frac{\sin l \cos(f \pm f')}{\sin l'' l'} \right\} \left\{ 1 + \frac{1}{16} e^2 \right\} + \frac{e^4}{32} \frac{\sin f^2}{\sin l''} \frac{\cos f}{\cos x} \sin(f - x)$$

$$\Sigma = a \sqrt{1 - ee} \sin_i I'' \sigma^l$$

Ponendo

$$\sigma = \frac{\Sigma}{a\sqrt{1-\cos \theta} \sin \theta}$$

$$\text{tang. } f' \equiv a\sqrt{1-e_e} \text{ tang. } \phi'$$

$$\text{tang. } X = \frac{\sin f'^3}{\sin (c \pm f)^2} \frac{\cos (c \pm f)^2}{\cos f^3}$$

obtinetur

$$l = \sigma - \frac{\frac{1}{4}e^2}{\sqrt{\frac{16}{(1-e\epsilon)^5}}}, \sigma \left\{ 1 - \frac{\sin \sigma \cos(\sigma + f'')}{\sigma \sin l''} \right\} \left\{ 1 + \frac{e^2}{2} \cos(\sigma + f')^2 \right\} - \frac{e^4}{32} \frac{\sin(\sigma + f')^2}{\sin l''} \frac{\cos(\sigma + f')}{\cos X} \sin(\sigma + f' - X)$$

$f \equiv f' + l$

$$\text{tang. } \bar{\varphi} = \frac{\text{tang. } \tilde{f}}{\sqrt{1 - \sec^2 f}}$$

Statuentes

$$\sigma = \frac{\sum}{a\sqrt{1-e_e} \sin i''}$$

$$\text{tang. } \tilde{f} = \text{tang. } \tilde{\phi} \sqrt{1-e^2}$$

$$\text{tang. } \mathfrak{X} = \frac{\sin. (\bar{f} - \sigma)^3}{\sin. \bar{f}^2 \cos. \bar{f}^2} \cos. (\bar{f} - \sigma)$$

adipiscemur

$$l = \sigma - \frac{\frac{e^2}{\lambda} \sigma'}{\sqrt{(1-e^2)^5}} \left\{ I - \frac{\sin \frac{1}{\lambda} \sigma \cos((2\tilde{f}) - \sigma)}{\sin \frac{1}{\lambda} \sigma} \right\} \left\{ I + \frac{e^2}{2} \cos((\tilde{f} - \sigma)^2) \right\} - \frac{e^4}{3^2} \frac{\sin \tilde{f}^2}{\sin \frac{1}{\lambda} \sigma} \frac{\cos \tilde{f}}{\cos \frac{1}{\lambda} \sigma} \sin((\tilde{f} - \tilde{X}))$$

unde, quin

$$f' = \tilde{f} - i$$

$$\tang. \varphi' = \frac{\tang. f'}{\sqrt{1 - ee}}$$

14.

Qvemadmodum series proxime expositæ ad illarum formam, de qvibus agitur in artt. 4, 5, reduci possint, id jam succinctim atqve uti digito est denotandum. Est generaliter, si in functione $f(u)$ ipsius u , pro u ponatur $u + i$

$$f(u+i) = f(u) + \frac{df(u)}{du} \cdot i + \frac{\frac{d^2f(u)}{d u^2}}{2!} \cdot i^2 + \dots$$

hinc

$$\text{arc.} \left\{ \tan. = \frac{a}{\sqrt{b+a^2}} \right\} = \text{arc.} \left\{ \tan. = \frac{a}{\sqrt{b}} \right\} - \frac{\frac{1}{2}a}{\sqrt{b(b+a^2)}} + \frac{\frac{1}{4}a^2}{\sqrt{b(b+a^2)}} \left\{ \frac{1}{2b} + \frac{1}{b+a^2} \right\}$$

proinde, quum

$$q = \sqrt{\frac{a^2 - p^2}{\rho^2 - \alpha'^2}} = \sqrt{1 - ee \sin \phi'^2} \sin \phi \left\{ \cos \phi^2 - \cos \phi'^2 \sin \alpha'^2 + \frac{ee}{1 - ee} \cos \phi^2 \cos \phi'^2 \cos \alpha'^2 \right\}^{-\frac{1}{2}}$$

$$= \sin \phi \left[\cos \phi^2 - \cos \phi'^2 \sin \alpha'^2 + \cos \phi'^2 (1 - \cos \phi'^2 \sin \alpha'^2) e^2 + \sin \phi'^4 (\cos \phi^2 - \cos \phi'^2 \sin \alpha'^2) e^4 - \cos \phi'^2 \sin \alpha'^2 \sin \phi'^2 e^2 + (1 + \sin \phi'^2) \cos \phi^2 \cos \phi'^2 \cos \alpha'^2 e^4 \right]^{-\frac{1}{2}}$$

ponatur

$$a = \sin \phi$$

$$b \equiv \cos_{\phi^2} - \cos_{\phi'^2} \sin_{\alpha'^2}$$

$$i = \cos\varphi^2 (1 - \cos\varphi'^2 \sin\alpha'^2) e^2 + \sin\varphi'^4 (\cos\varphi^2 - \cos\varphi'^2 \sin\alpha'^2) e^4 - \cos\varphi'^2 \sin\alpha'^2 \sin\varphi'^2 e^2 + (1 + \sin\varphi'^2) \cos\varphi^2 \cos\varphi'^2 \cos\alpha'^2 e^4$$

atque exinde evolvatur

$$\text{arc.} (\tan. \equiv q) = v - \frac{1}{2} e^2 \left\{ \sin. \gamma^2 \sin. v \cos. v + \cos. \gamma^2 \cos. v^{1/2} \tan. v + \frac{3}{4} e^2 \sin. \gamma^4 \sin. v \cos. v \right\} \\ - \frac{1}{4} e^4 \left\{ \tan. v \cos. v^{1/2} \cos. \gamma^2 (1 + \sin. v^{1/2} (1 + \sin. \gamma^2)) - \frac{\sin. v \cos. v^{1/4}}{2 \cos. v^3} \cos. \gamma^4 - \sin. v \cos. v^3 \sin. \gamma^4 \right\}$$

deinceps,

$$\text{ut pote } \frac{d}{a} q = \cos. \phi' \sin. \alpha' \sin. \phi \left\{ \cos. \phi^2 - \cos. \phi'^2 \sin. \alpha'^2 + (1 + ee) e^2 \cos. \phi^2 \cos. \phi'^2 \cos. \alpha'^2 \right\} - \frac{1}{2}$$

ponatur

$$a = \cos. \phi' \sin. \alpha' \sin. \phi$$

$$b = \cos. \phi^2 - \cos. \phi'^2 \sin. \alpha'^2$$

$$i = (1 + ee) ee \cos. \phi^2 \cos. \phi'^2 \cos. \alpha'^2$$

et similiter evolvatur

$$\text{arc.} \left\{ \tan. \equiv \frac{d}{a} \cdot q \right\} = \mu - \frac{1}{2} e^2 \cos. v^{1/2} \tan. \mu - \frac{e^4}{4} \cos. v^{1/2} (1 + \sin. v^{1/2}) \tan. \mu + \frac{e^4}{8} \cos. v^{1/4} \frac{\tan. \mu}{\cos. \mu^2}$$

porro explicetur per formulam binomialem

$$\text{et } \frac{a^2 + j^2}{a^2} = 1 + \cos. \gamma^2 + e^2 \cos. \gamma^2 \sin. v^{1/2} \sin. \gamma^2$$

$$\text{et } \frac{a^2 - j^2}{a^2} \frac{q}{1+q^2} = \sin. \gamma^2 \left\{ \sin. v \cos. v + \frac{e^2}{2} \sin. \gamma^2 \sin. v \cos. v - \frac{e^2}{2} \cos. \gamma^2 \cos. v^{1/2} \tan. v - e^2 \sin. v \cos. v (1 - \sin. \gamma^2 \sin. v^2) \right\}$$

deinde valores ita erutos in aequationibus art. II substituentur, pluribus confectis transformationibus, tandem adseqvemur

$$\omega = \delta - \frac{e^2}{2} \cos. \gamma^2 \lambda \left\{ 1 + \frac{e^2}{2} \sin. \gamma^2 \sin. v^{1/2} + \frac{1 + \cos. \gamma^2}{4} \right\} - \frac{e^2}{2} \cos. \phi'^2 \frac{\cos. \mu'}{\cos. \mu} \sin. \delta \left\{ 1 + \frac{e^2}{2} (\sin. v^{1/2} + \sin. \gamma^2) \right\} \\ + \frac{3}{16} e^4 \sin. \gamma^2 \cos. v \sin. \lambda \cos. (v + v') + \frac{e^4}{8} \cos. v^{1/4} \left\{ \frac{\tan. \mu}{\cos. \mu^2} - \frac{\tan. \mu'}{\cos. \mu'^2} \right\}$$

$$\Sigma = a \lambda \left\{ 1 - \frac{e^2}{4} (1 + \cos. \gamma^2) - \frac{e^4}{8} (\cos. \gamma^2 (1 + 2 \sin. \gamma^2 \sin. v^{1/2}) + \frac{3}{8} \sin. \gamma^4) \right\}$$

$$- \frac{3}{4} a e^2 \sin. \gamma^2 \sin. \lambda \cos. (v + v') \left\{ 1 - \frac{e^2}{16} (1 + 7 \cos. \gamma^2) \right\}$$

$$- \frac{1}{2} a e^2 \cos. \gamma^2 \frac{\cos. v'}{\cos. v} \sin. \lambda \left\{ 1 + \frac{e^2}{2} (\sin. \gamma^2 + \sin. v^{1/2} (1 + \sin. \gamma^2)) \right\}$$

$$- \frac{15}{32} a e^4 \sin. \gamma^4 (\sin. v^3 \cos. v - \sin. v^{1/3} \cos. v') + \frac{1}{8} a e^4 \cos. \gamma^4 \cos. v'^4 \left(\frac{\sin. v}{\cos. v^3} - \frac{\sin. v'}{\cos. v'^3} \right)$$

Quæ series in art. seqvente ad formam commodiorem reducentur, ibidemque cæteræ exponentur ab iisdem derivandæ.

Aeqvationes conditionales, in articulo hoc brevitatis gratia omisfas, ex artt. 4, 5 esse supplendas, monitu vix eget.

Ponendo

$$\tan \eta = \frac{\tan \mu' \cos \mu^2}{\cos \mu'^2}$$

habetur

$$\begin{aligned} \omega &= \delta - \frac{\frac{1}{4}ee}{\sqrt{1-ee}} \cos \gamma \lambda \left\{ i - \frac{e^2}{2} \sin \gamma^2 \cos(60^\circ + \nu') \cos(60^\circ - \nu') \right\} \\ &\quad - \frac{1}{2}ee \frac{\cos \phi'^2}{\sin i''} \frac{\cos \mu'}{\cos \mu} \sin \delta \left\{ i + \frac{e^2}{2} (\sin \nu'^2 + \sin \gamma^2) \right\} \\ &\quad + \frac{3}{16} \frac{e^4}{\sin i''} \cos \gamma \sin \gamma^2 \sin \lambda \cos(\nu + \nu') + \frac{e^4}{8} \frac{\cos \nu'^4}{\sin i''} \frac{\sin(\mu - \eta)}{\cos \mu^2 \cos \eta} \dots \quad (1) \end{aligned}$$

Ponendo

$$\tan H = \tan \mu' \frac{(\cos \omega + \mu')^2}{\cos \mu'^2}$$

obtinetur

$$\begin{aligned} \delta &= \omega + \frac{\frac{1}{4}ee}{\sqrt{1-ee}} \cos \gamma \left\{ N - \nu' \right\} \left\{ i - \frac{e^2}{2} \sin \gamma^2 \cos(60^\circ + \nu') \cos(60^\circ - \nu') \right\} \\ &\quad + \frac{e^2 \cos \phi'^2 \cos \mu' \sin \omega}{2 \sin i'' \cos(\omega + \mu')} \left\{ i + \frac{e^2}{2} (\sin \nu'^2 + \sin \gamma^2) \right\} \\ &\quad - \frac{3}{16} \frac{e^4}{\sin i''} \cos \gamma \sin \gamma^2 \sin(N - \nu') \cos(N + \nu') - \frac{e^4 \cos \nu'^4 \sin(\omega + \mu' - H)}{8 \sin i'' \cos(\omega + \mu')^3 \cos H} \\ &\quad + \frac{e^4}{4} \cos \gamma (N - \nu') \frac{\cos N^2}{\cos(\omega + \mu')^2} \left\{ i + \frac{\cos \nu'^2}{\cos N^2} \right\} \left\{ i + \frac{\cos \phi'^2 \cos \mu' \sin \omega}{\sin i'' (N - \nu') \cos \gamma \cos(\omega + \mu')} \right\} \dots \quad (2) \end{aligned}$$

Ponatur

$$\tan \xi = \frac{\sin \nu'^3 \cos \nu'}{\sin \nu^2 \cos \nu^2}$$

$$\tan \varepsilon = \tan \nu' \frac{\cos \nu^2}{\cos \nu'^2}$$

obtinetur

$$\begin{aligned} \sigma &= \lambda \left\{ i + \frac{e^2}{4} \sin \gamma^2 (i + e^2 \cos \nu'^2 \cos \gamma^2 + \frac{1}{16} e^2 \sin \gamma^2) \right\} \\ &\quad - \frac{3}{4} \frac{e^2}{\sin i''} \sin \gamma^2 \sin \lambda \cos(\nu + \nu') \left\{ i + \frac{7}{16} e^2 \sin \gamma^2 \right\} \\ &\quad - \frac{e^2 \cos \gamma^2 \cos \nu'}{2 \sin i'' \cos \nu} \sin \lambda \left\{ i + \frac{e^2}{2} (i + \sin \gamma^2)(i + \sin \nu'^2) \right\} \\ &\quad - \frac{15}{32} \frac{e^4 \sin \gamma^4}{\sin i'' \cos \xi} \sin \nu^2 \cos \nu \sin(\nu - \xi) + \frac{e^4 \cos \nu'^4 \cos \nu'^2 \sin(\nu - \sigma)}{8 \sin i'' \cos \varepsilon \cos \nu^3} \dots \quad (3) \end{aligned}$$

Stituendo

$$\text{tang. } X = \frac{\sin \nu^2 \cos \nu^2}{\sin (\sigma + \nu)^2 \cos (\sigma + \nu)^2}$$

$$\text{tang. } E = \text{tang. } \nu^2 \frac{\cos (\sigma + \nu)^2}{\cos \nu^2}$$

prodit

$$\lambda = \sigma \left\{ 1 - \frac{e^2}{4} \sin \gamma^2 (1 + e^2 \cos \nu^2 \cos \gamma^2 + \frac{1}{16} e^2 \sin \gamma^2) \right\}$$

$$+ \frac{e^2}{4} \sin \nu^2 \sin \gamma^2 \sin \sigma \cos (\sigma + 2\nu^2) \left\{ 1 + \frac{7}{16} e^2 \sin \gamma^2 \right\}$$

$$+ \frac{e^2 \cos \gamma^2 \cos \nu^2 \sin \sigma}{2 \sin \nu^2 \cos (\sigma + \nu^2)} \left\{ 1 + \frac{e^2}{2} (1 + \sin \gamma^2) (1 + \sin \nu^2) \right\}$$

$$+ \frac{15}{32} \frac{e^4 \sin \gamma^4}{\sin \nu^2 \cos X} \sin (\sigma + \nu^2)^2 \cos (\sigma + \nu^2) \sin (\sigma + \nu^2 - X) - \frac{e^4 \cos \gamma^4 \cos \nu^4 \sin (\sigma + \nu^2 - E)}{8 \sin \nu^2 \cos E \cos (\sigma + \nu^2)^3}$$

$$+ \frac{e^4 \sin \gamma^4}{8 \sin \nu^2} \left\{ 1 - 3 \sin (\sigma + \nu^2)^2 + \cot \nu^2 \frac{\cos \nu^2}{\cos (\sigma + \nu^2)} \right\} \left\{ \begin{array}{l} \sin \nu^2 \cdot \sigma + 3 \sin \sigma \cos (\sigma + 2\nu^2) \\ \cos \nu^2 \end{array} \right\} + 2 \cot \nu^2 \sin \sigma \frac{\cos \nu^2}{\cos (\sigma + \nu^2)} \dots \dots \dots (4)$$

$$\omega = \bar{\delta} - \frac{\frac{1}{2} e^2}{\sqrt{1-e^2}} \cos \bar{\phi} \cdot \bar{\lambda} \left\{ 1 + \frac{3}{8} e^2 \sin \bar{\phi}^2 \right\} - \frac{e^4}{32} \frac{e^4}{\sin \nu^2} \cos \bar{\phi} \sin \bar{\phi}^2 \sin 2\bar{\lambda} \dots \dots \dots (5)$$

$$\tilde{\delta} = \omega + \frac{\frac{1}{2} ee}{\sqrt{1-ee}} \cos \bar{\phi} \cdot \bar{\lambda} \left\{ 1 + \frac{3}{8} e^2 \sin \bar{\phi}^2 \left(1 + \frac{4}{3} \cot \bar{\phi}^2 \frac{\cos \bar{\lambda}^2}{\cos \omega^2} \right) \right\} + \frac{e^4}{32} \frac{e^4}{\sin \nu^2} \cos \bar{\phi} \sin \bar{\phi}^2 \sin 2\bar{\lambda} \dots \dots \dots (6)$$

$$\tilde{\sigma} = \omega + \frac{\frac{1}{2} ee}{\sqrt{1-ee}} \cos \bar{\phi} \cdot \bar{\lambda} \left\{ 1 + \frac{e^2}{2} \left(1 - \frac{\sin \bar{\phi}^2}{4} - \sin \bar{\phi}^2 \tan \bar{\lambda} \sin \nu^2 \cdot \bar{\lambda} \right) \right\} + \frac{3}{32} \frac{e^4}{\sin \nu^2} \cos \bar{\phi} \sin \bar{\phi}^2 \sin 2\bar{\lambda} \dots \dots \dots (7)$$

$$\tilde{\sigma} = \bar{\lambda} \left\{ 1 + \frac{e^2}{4} \sin \bar{\phi}^2 (1 + \frac{1}{16} e^2 \sin \bar{\phi}^2) \right\} + \frac{e^2}{4} \frac{e^2}{\sin \nu^2} \sin \bar{\phi}^2 \sin \bar{\lambda} \cos \bar{\lambda} \left\{ 1 + \frac{7}{16} e^2 \sin \bar{\phi}^2 \left(1 + \frac{\cos \bar{\lambda}^2}{0,7} \right) \right\} \dots \dots \dots (8)$$

$$\tilde{\lambda} = \bar{\sigma} \left\{ 1 - \frac{e^2}{4} \sin \bar{\phi}^2 (1 + \frac{1}{16} e^2 \sin \bar{\phi}^2) [8 \sin \sigma^2 - 1] \right\} - \frac{3}{4} \frac{e^2}{\sin \nu^2} \sin \bar{\phi}^2 \sin \bar{\sigma} \cos \bar{\sigma} \left\{ 1 + \frac{e^2}{16} \sin \bar{\phi}^2 (1 + 14 \sin \bar{\sigma}^2) \right\} \dots \dots \dots (9)$$

$$\tilde{\lambda} = \bar{\sigma} \left\{ 1 - \frac{e^2}{4} \sin \bar{\phi}^2 \left[1 - \frac{e^2}{16} \sin \bar{\phi}^2 (3 + 8 \sin \nu^2 \cdot \sigma \tan \sigma + 24 \sin \sigma^2) \right] \right\} - \frac{3}{4} \frac{e^2}{\sin \nu^2} \sin \bar{\phi}^2 \sin \bar{\sigma} \cos \bar{\sigma} \left\{ 1 - \frac{e^2}{16} \sin \bar{\phi}^2 (10 \sin \sigma^2 - 1) \right\} \dots \dots \dots (10)$$

Posito

$$\text{tang. } \xi = \frac{\sin \bar{\phi}^2 \cos \bar{\phi}^2}{\sin \bar{\phi}^2 \cos \bar{\phi}^2}$$

prodit

$$\sigma = \lambda \left\{ 1 + \frac{e^2}{4} \left(1 + \frac{1}{16} e^2 \right) \right\} - \frac{3}{4} \frac{e^2}{\sin i''} \sin \lambda \cos(\phi + \varphi') \left[1 + \frac{7}{16} e^2 \right] - \frac{15 e^4 \sin \tilde{\phi}^2 \cos \tilde{\phi} \sin(\tilde{\phi} - \xi)}{32 \sin i'' \cos \xi} \dots (11)$$

Posito

$$\tan g. X = \frac{\sin \phi'^3 \cos \phi'}{\sin (\sigma + \varphi')^2 \cos (\sigma + \varphi')^2}$$

oritur

$$\begin{aligned} \lambda = \sigma &= \frac{e^2}{4} \sqrt{\frac{x \epsilon}{(1-e^2)^3}} \cdot \sigma \left\{ 1 - \frac{3 \sqrt{(1-e^2)^3} \sin \sigma \cos(\sigma + 2\varphi')}{\sin i'' \sigma} \right\} \left[1 + \frac{3}{2} e^2 \cos(\sigma + \varphi')^2 \right] \\ &\quad + \frac{15}{32} \frac{e^4}{\sin i''} \frac{\cos(\sigma + \varphi')}{\cos X} \sin(\sigma + \varphi')^2 \sin(\sigma + \varphi' - X) \dots \dots \dots (12) \end{aligned}$$

Statuendo

$$\tan g. \tilde{\rho} = \frac{\sin(\tilde{\phi} - \sigma) \cos(\tilde{\phi} - \sigma)}{\sin \tilde{\phi}^2 \cos \tilde{\phi}^2}$$

prodit

$$\begin{aligned} \lambda = \sigma &= \frac{e^2}{4} \sqrt{\frac{x \epsilon}{(1-e^2)^3}} \cdot \sigma \left\{ 1 - \frac{3 \sqrt{(1-e^2)^3} \sin \sigma \cos(\tilde{\phi} - \sigma)}{\sin i'' \sigma} \right\} \left[1 + \frac{3}{2} e^2 \cos(\tilde{\phi} - \sigma)^2 \right] \\ &\quad + \frac{15}{32} \frac{e^4}{\sin i''} \frac{\cos \tilde{\phi}}{\cos \tilde{\rho}} \sin \tilde{\phi}^2 \sin(\tilde{\phi} - \tilde{\rho}) \dots \dots \dots \dots \dots (13) \end{aligned}$$

16.

Si quis forte prolixiores illas articulorum 12, 13, 15 series per calculum numericum confirmatas velit, jam haec afferentur.

In ellipsoide, cui

$$\log. \alpha = 6,514754624$$

$$\log. e = 8,904485625$$

Datis

$$\varphi' = 36^\circ 32' 1'', 00 \quad \varphi = 59^\circ 56' 23'', 00 \quad \omega = 36^\circ 35' 45'', 00$$

Inventum

$$\alpha' = 33^\circ 17' 37'', 72 \text{ per form. 2.c art. 12} \quad \alpha' = 33^\circ 17' 37'', 72 \text{ per form. 2.c art. 15}$$

Datis

$$\varphi' = 36^\circ 32' 1'', 00 \quad \varphi = 59^\circ 56' 23'', 00 \quad \alpha' = 33^\circ 17' 37'', 72$$

Inventum

$$\omega = 36^\circ 35' 44'', 96 \text{ per form. 1.a art. 12} \quad \omega = 36^\circ 35' 44'', 96 \text{ per form. 1.a art. 15}$$

Datis

$$\varphi' = 36^\circ 32' 1'',00 \quad \varphi = 59^\circ 56' 23'',00 \quad \alpha' = 33^\circ 17' 37'',72$$

Inventum

$$\Sigma = 18867 \text{ tois. per form. } 3 \text{. a art. } 12 \quad \Sigma = 18867 \text{ tois. per form. } 3 \text{. a art. } 15$$

Datis

$$\varphi' = 36^\circ 32' 1'',00 \quad \alpha' = 33^\circ 17' 37'',72 \quad \Sigma = 18867 \text{ tois.}$$

Inventum

$$\varphi = 59^\circ 56' 23'',00 \text{ per form. } 4 \text{. a art. } 12 \quad \varphi = 59^\circ 56' 23'',07 \text{ per form. } 4 \text{. a art. } 15$$

Datis

$$\varphi = 59^\circ 56' 23'',00 \quad \omega = 36^\circ 35' 45'',00$$

Inventum

$$\varphi = 65^\circ 5' 3'',77 \text{ per form. } 7 \text{ art. } 13 \quad \varphi = 65^\circ 5' 3'',77 \text{ per form. } 7 \text{ art. } 15$$

Datis

$$\varphi = 65^\circ 5' 3'',77 \quad \varphi = 59^\circ 56' 23'',00$$

Inventum

$$\omega = 36^\circ 35' 45'',00 \text{ per form. } 5 \text{ art. } 13 \quad \omega = 36^\circ 35' 45'',00 \text{ per form. } 5 \text{ art. } 15$$

Datis

$$\varphi = 65^\circ 5' 3'',77 \quad \omega = 36^\circ 35' 45'',00$$

Inventum

$$\varphi = 59^\circ 56' 23'',00 \text{ per form. } 6 \text{ art. } 13 \quad \varphi = 59^\circ 56' 23'',00 \text{ per form. } 6 \text{ art. } 15$$

Datis

$$\varphi = 65^\circ 5' 3'',77 \quad \varphi = 59^\circ 56' 23'',00$$

Inventum

$$\Sigma = 994508,7 \text{ tois. per form. } 8 \text{ art. } 13 \quad \Sigma = 994508,5 \text{ tois. per form. } 8 \text{ art. } 15$$

Datis

$$\varphi = 65^\circ 5' 3'',77 \quad \Sigma = 994508,6 \text{ tois.}$$

Inventum

$$\varphi = 59^\circ 56' 23'',00 \text{ per form. } 9 \text{ art. } 13 \quad \varphi = 59^\circ 56' 23'',00 \text{ per form. } 9 \text{ art. } 15$$

Datis

$$\varphi = 59^\circ 56' 23'',00 \quad \Sigma = 994508,6 \text{ tois.}$$

Inventum

$$\varphi = 65^\circ 5' 3'',77 \text{ per form. } 10 \text{ art. } 13 \quad \varphi = 65^\circ 5' 3'',77 \text{ per form. } 10 \text{ art. } 15$$

Datis

$$\phi' = 36^\circ 32' 1'', 00 \quad \bar{\phi} = 65^\circ 5' 3'', 77$$

Inventum

$$\Sigma = 1629116,3 \text{ tois. per form. 11 art. 13} \quad \dot{\Sigma} = 1629116,4 \text{ tois. per form. 11 art. 15}$$

Datis

$$\phi' = 36^\circ 32' 1'', 00 \quad \dot{\Sigma} = 1629116,3 \text{ tois.}$$

Inventum

$$\bar{\phi} = 65^\circ 5' 3'', 76 \text{ per form. 12 art. 13} \quad \bar{\phi} = 65^\circ 5' 3'', 76 \text{ per form. 12 art. 15}$$

Datis

$$\bar{\phi} = 65^\circ 5' 3'', 77 \quad \dot{\Sigma} = 1629116,3 \text{ tois.}$$

Inventum

$$\phi' = 36^\circ 32' 1'', 01 \text{ per form. 13 art. 13} \quad \phi' = 36^\circ 32' 1'', 00 \text{ per form. 13 art. 15.}$$

Monendum est, leviores qvæ observantur discrepantias vitio tabularum omnino non deberi, quum ampliores Vlacci, qvoties opus fuerit, adhibitæ sint; neque tamen illas majoris esse momenti, qvam ut ex inevitabili serierum errore explicari possint.

17.

Periclitandum est tentamina circa triangulum ellipsoidicum, nulli adstrictum conditioni, solvendum exponere.

Qvod anteqvam aggrediamur, termini necessarii presse, qvoad ejus fieri posse, præmittantur.

Designentur latitudines primi, secundi et tertii puncti angularis ϕ' ϕ'' ϕ'''
differentiae longitudinum, ratione primi et secundi, secundi et tertii,

primi et tertii puncti angularis ω' ω'' ω'''
latera, sive lineæ brevisimæ inter primum et secundum, secundum

et tertium, primum et tertium punctum angulare in superficie ellipsoidis ductæ Σ' Σ'' Σ'''
anguli his oppositi ad ordinem β' β'' β'''

Deinde meridianos per tertium qvodqve trianguli punctum angulare fingendo, generaliter dicantur:

angulus, qvo latus meridianum intersecat perpendiculus a puncto angulari in curvam meridiani demissa Σ
 latitudo pedis, cui perpendiculus insistit ϕ
 arcus meridiani a pede et punto angulari interceptus Σ
 angulus, qvo latus in perpendicularem inclinatur ψ

Quantitates *specialiores* singulis his quinque comprehensæ, sicuti facile ab invicem discernuntur, si modo meridianus et latus, quibus in uno punto angulari communis est intersectio, et a quibus specialiores eadem quantitates dependent, notentur, ita haud incommodè per generaliores illas designantur notas, dupli tantummodo cifra adjuncta. Altera nimis cifra, notæ generaliori a vertice dextrorum affixa, convenienter indici, qvo nota lateris, ad quod quantitas specialior spectat, jam insignita est; altera, a calce sinistrorum suffixa, indicabit, quod sit meridianus, ad quem eadem quantitas specialior referatur.

Exempli causa:
 $\text{m}\bar{\phi}''$ est nota, qua designatur latitudo pedis, cui insistit perpendiculus ad latus secundum pariter ac meridianum tertium spectans.

Porro præparentur singuli casus per tabulam calci hujus libelli subjunctam. Per multæ cernuntur elementorum complexiones, cum terniones, tum biniones, in hac tabula depositæ. Quantumvis magnus earundem sit numerus, consilio tamen necessario fere duplicari debuisset, nisi præsidio notarum generalium adjuti essemus. Sed harum usu multæ indefinitæ complexiones ortæ sunt, haud quidem prorsus indefinitæ, quum singula cujusque elementa ad idem latus spectent, at potius, in proprio vocis sensu, ambiguæ, quippe in ambiguo relinquitur, ad quemnam duorum meridianorum, quos idem intersecat latus, pertineant. *Definitæ* igitur audient, ubi hoc decretum sit.

Jam quoties ex *definitis* illis ternionibus et binionibus *unus quilibet* ternio unacum *uno quilibet* binione datus sit, solutione trianguli ellipsoidici potiemur; ne vero longi simus, subsystemus, ut primum latitudines, differentiæ longitudinum, latera et anguli innescunt.

Totum solutionis actum in quatuor negotia partiemur.

Negotium primum. Solvetur triangulum meridiano-rectangulum, quod per ternionem datum determinatur, adeo ut ϕ' , ϕ'' , ω' , Σ' et alterutrum α' sive α' sive α''

innoscant. Qvod ut facilius præstetur, ternioni appositus est idem numerus, qvo in artt. 6-9 casus noster unacum solutione sua notatus legitur; modo observetur, indicio prioris numeri utendum esse, si ternio datus ad meridianum primum spectet, posterioris, si ad secundum. Ex φ' φ'' et alterutro α' aliud α'' invenitur propter form. 14.

Negotium secundum aliud erit, si binio datus in 35 priores incidit, aliud, si in posteriores 35. De unoqvoqve singulatim dicetur.

Si binio datus in *priores* 35 incidit, solvetur triangulum meridiano-rectangulum, qvod per φ'' et binionem datum determinatur, donec deteguntur $\varphi''' \omega'' \Sigma''$ et alterutrum α'' sive „ α'' sive „ α'' “. Numerus binioni appositus istum in artt. 6-9 tractatum casum indigitabit, ex qvo typus solutionis qvæsitæ depromi posit; prior quidem, si binio datus meridianum secundum spectet, posterior, si tertium. Ex $\varphi'' \varphi'''$ et alio α'' aliud propter form. 14 obtinebitur. Qvod si binio datus in septem posteriores horum jam dictorum 35 inciderat, ante omnia sumetur differentia $360^\circ - (\beta''' + \alpha'')$, ut inde proveniat „ α'' “, qvo operatio jam generaliter descripta perfici potest.

Si binio datus in *posteriores* 35 incidit, est triangulum meridiano-rectangulum, qvod per binionem datum et φ' determinatur, ita solvendum, ut $\varphi''' \omega''' \Sigma'''$ et alterutrum α''' sive „ α''' sive „ α''' “ eruantur, numero binioni affixo easum in artt. 6-9 indicante, ad cujus instar solutio casus præsentis confici posfit; priori quidem, si binio datus ad meridianum primum referatur; posteriori, si ad tertium. Tandem ex $\varphi' \varphi'''$ et altero α''' alterum per form. 14 obtineri potest.

Si binio ad septem posteriores horum 35 pertinet, primum, α' angulo dato β'' addi debet, unde oritur qvantitas „ α''' “, qva calculus jam generaliter descriptus perfici potest.

Negotium tertium anteqvam ordiamur, iterum memores esse debemus, utrum prioribus 35 an posterioribus 35 binio datus annumeretur.

Si obtineat illud, formabitur $\omega''' = \omega' + \omega''$ et ex $\varphi' \varphi''' \omega'''$ juxta casum I in art. 6 invenientur Σ''' et alterutrum α''' sive „ α''' sive „ α''' “ atqve ex altero α''' una cum $\varphi' \varphi'''$ alterum juxta form. 14 investigabitur; si hoc, sistetur $\omega''' = \omega''' - \omega'$, nec non ex $\varphi''' \varphi'' \omega''$ juxta casum I art. 6 conseqvemur Σ''' et alterutrum α'' sive „ α'' sive „ α'' “, ex qvo, φ''' et φ'' alterum α'' propter form. 14 habebitur.

Negotium quartum æquationibus his absolyetur

$$\beta' = \alpha''' - \alpha''$$

$$\beta'' = \alpha''' - \alpha'$$

$$\beta''' = 360^\circ - (\alpha' + \alpha'')$$

Monendum insuper, commoditati aliquantum eo consultum fore, qvod primum omnium, qvotiescunqve ad negotia demandata aggrediamur, compositiores notæ, qvibus insigniatur casus propositus, ad simplices illas in triangulo primario artt. 6-9 obvias transferantur; deinde autem, singulo qvovis confecto negotio, rite ac legitime, ut omni occurratur confusione, in statum pristinum restituantur.

I8.

Articuli præcedentis illustrandi gratia, exemplum ab ipsa tellure petitum exhibebitur. Illud nominatim triangulum ellipsoidicum, qvod per Gades, Petroburgum et Bombay transit, est completa subducta ratione numerica solvendum.

Ecce data

$$\varphi' = \text{latitudo Gadium} \dots \dots \dots \dots \dots \dots \quad 36^\circ 32' 1'',0000$$

$$\alpha' = \text{angulus, qvo latus, a Gadibus ad Petroburgum excurrens, meidianum Gaditanum intersecat} \dots \dots \dots \quad 33^\circ 17' 37'',7248$$

$$\psi' = \text{angulus, qvem latus jam dictum et perpendicularis a Petroburgo in meridianum Gaditanum demissa, inter cludunt} \dots \dots \dots \dots \dots \dots \quad 61^\circ 9' 26'',2268$$

$$\Sigma'' = \text{latus a Petroburgo ad Bombay descendens} \dots \dots \dots \quad 2907413,955 \text{ tois.}$$

$$\alpha'' = \text{angulus, qvo hocce latus a meridiano Petroburgi deflectitur} \dots \dots \dots \dots \dots \dots \quad 124^\circ 25' 32'',0883$$

Ut via ad ipsum solutionis actum præsternatur,

e Commercio litterario clar. de Zach (Vol. XXVI pag. 59) laudantur

$$a = \text{radius æquatoris terrestris} \dots \dots \dots \dots \dots \dots \quad 3271558 \text{ tois.}$$

$$b = a\sqrt{1-e^2} = \text{semiaxis terrestris} \dots \dots \dots \dots \dots \dots \quad 3261005 =$$

et sistitur valor differentialis statim adhibendus

$$d\omega = d\psi : \left\{ -\frac{\cot. \alpha \sin. \alpha^2}{\cos. \tilde{d} \sin. \tilde{d}} + \frac{\sin. \tilde{f} \cos. \tilde{f}}{\cos. f \cos. \tilde{d}} + \frac{\tan. \alpha}{\tan. \alpha} \sin. f - \sin. f \right\}$$

nec non id præmonetur, minusculas lingvæ vernaculæ litteras, qvo-simplicius complexæ designentur quantitates, modo per se perspicuo freqventer esse adhibendas.

Negotium primum.

Translatis φ' , α' , ψ' , φ'' , ω' , Σ' , n'
in φ' , α' , ψ , φ , ω , Σ , n

$$\log. \cos. \psi = 9,6834134$$

$$C. \log. \sin. \alpha' = 0,2604810$$

$$\log. \cos. \bar{\varphi} - \varphi' = 9,9438944$$

$$\varphi - \varphi' = 28^\circ 30' 3'',59$$

$$\varphi' = 36^\circ 32' 1'',00$$

$$\bar{\varphi} = 65^\circ 2' 4'',59$$

$$C. \log. \cos. \bar{\varphi} = 0,3746148$$

$$\log. \sin. \bar{\varphi} - \varphi' = 9,6786769$$

$$\log. \tang. \alpha' = 9,8173820$$

$$\log. \tang. \omega = 9,8706737$$

$$\omega = 36^\circ 35' 32'',82$$

valor suppositus ω per trig. sphæric. deductus.

$$\log. \sqrt{1-e^2} = 9,998596786$$

$$\log. \tang. \varphi' = 9,869741596$$

$$\log. \tang. f' = 9,868338382$$

$$\log. \sin. \alpha' = 9,739518986$$

$$\log. \cos. f' = 9,905486317$$

$$\log. \cos. g = 9,645005303$$

$$\log. \tang. \alpha' = 9,817381990$$

$$\log. \sin. f' = 9,773824699$$

$$\log. \tang. m' = 9,591206689$$

$$m' = 21^\circ 18' 43'',6668$$

$$\log. \tang. f' = 9,868338382$$

$$C. \log. \cos. \alpha' = 0,077863004$$

$$\log. \tang. n' = 9,946201386$$

$$n' = 41^\circ 27' 36'',9315$$

$$\log. \tang. (\omega + m') = 0,2026027$$

$$C. \log. \cos. g = 0,3549947$$

$$\log. \tang. N = 0,5575974$$

$$N = 74^\circ 31' 11'',60$$

$$n' = 41^\circ 27' 36'',93$$

$$N - n' = 33^\circ 3' 34'',67$$

$$= 119014'',67$$

$$\log. \cos. (\omega + m') = 9,72537$$

$$C. \log. \cos. N = 0,57365$$

$$\log. \cos. g = 9,64501$$

$$\log. \frac{r}{2} = 9,69897$$

$$\log. \tang. q = 9,64300$$

$$C. \log. \cos. q = 0,03836$$

$$\log. \cos. N = 9,42635$$

$$C. \log. \cos. (\omega + m') = 0,27463$$

$$\log. e : \sqrt{2} = 8,75397$$

$$\log. \tang. r = 8,49331$$

$$C. \log. \cos. r^2 = 0,0004210$$

$$\log. N - n' = 5,0756005$$

$$\log. \cos. g = 9,6450053$$

$$\log. \frac{1}{2} e^2 : \sqrt{1-e^2} = 7,5082880$$

$$\log. 169,5566 = 2,2293148$$

$$2'49'',5566$$

$$\log. \frac{e^4}{\sqrt{2}} : \sin. 1'' = 9,72824$$

$$\log. \cos. g = 9,64501$$

$$\log. \sin. g^2 = 9,90580$$

$$\log. \sin. N - n' = 9,73681$$

$$\log. \cos. N + n' = 9,64153$$

$$\log. 0'',0454 = 8,65739$$

$$\begin{array}{r} \text{velocitas obliqua } \omega = 36^\circ 35' 32'', 8200 \\ \text{velocitas obliqua } \omega + 249,5566 \\ - \quad \quad \quad 0,0454 \\ \hline d = 36^\circ 38' 22'', 3312 \end{array}$$

$$\begin{array}{r} m' = 21 18 43,6668 \\ m = 57^\circ 57' 5'', 9980 \end{array}$$

$$\log. \sin. m = 9,928191340$$

$$\log. \tan. g = 0,307895544$$

$$\log. \tan. f = 0,236086884$$

unde valores f et ϕ supposito ω accommodatos assequemur.

$$\log. \tan. f = 0,23608689$$

$$C. \log. \cos. \omega = 0,0953407$$

$$\log. \tan. F = 0,3314276$$

$$\log. \cos. F = 9,6258660$$

$$\log. \tan. \omega = 9,8706737$$

$$\log. \tan. \bar{\mathcal{L}} = 9,4965397$$

$$\log. \tan. \bar{\mathcal{L}} = 4,79729$$

$$\log. 0,8 \sin. 1'' = 4,58866$$

$$C. \log. \cos. F^2 = 0,74827$$

$$\log. \cos. m^2 = 9,63076$$

$$\log. \cos. m = 9,81538$$

$$\log. \sin. m = 9,87895$$

$$\log. \cos. F = 9,62587$$

$$\log. e : \sqrt{1,6} = 8,80242$$

$$\log. \tan. n = 8,30724$$

$$C. \log. \cos. n^2 = 0,0001788$$

$$\log. \bar{\mathcal{L}} = 4,7972905$$

$$\log. \cos. F = 9,6258660$$

$$\log. \frac{e^2}{2} : \sqrt{1-e^2} = 7,5082880$$

$$\log. 85'',4325 = 1,9316233$$

$$1'25'',4325$$

$$\begin{array}{r} \log. \frac{e^4}{16} : \sin. 1'' = 9,72824 \\ \log. \cos. F = 9,62587 \\ \log. \sin. F^2 = 9,91458 \\ \log. \sin. \bar{\mathcal{L}} = 9,47616 \\ \log. \cos. \bar{\mathcal{L}} = 9,97961 \end{array}$$

$$\begin{array}{r} \log. 0,0530 = 8,72446 \\ \log. \cos. \omega = 36^\circ 35' 32'', 8200 \\ \log. \cos. \omega + 1'25'', 4325 \\ - \quad \quad \quad 0,0530 \\ \hline d = 36^\circ 36' 58'', 1995 \end{array}$$

$$C. \log. \cos. d = 0,095474216$$

$$\log. \tan. f = 0,236086884$$

$$\log. \tan. f = 0,331561100$$

unde valores f et ϕ supposito ω accommodati erui posunt.

$$\log. \cos. f = 9,625756324$$

$$C. \log. \cos. f = 0,299177335$$

$$\log. \sin. \alpha = 9,924933659$$

$$\alpha = 57^\circ 16' 26'', 8125$$

valor ipsius α supposito ω accommodatus.

$$\log. \cos. g = 9,645005303$$

$$C. \log. \cos. f = 0,299177335$$

$$\log. \sin. \alpha = 9,944182638$$

$\alpha = 118^\circ 25' 51'', 1011$
valor ipsius α supposito ω accommodatus.

$$\begin{array}{r} \psi = \alpha - \omega = 61^\circ 9' 24'', 2886 \\ \psi \text{ datum} = 61 9 26,2268 \end{array}$$

$d\psi = 1'', 9382$
differentia inter ψ datum et ψ computatum.

$$\log. \cot. \alpha = 9,7335137_n$$

$$\log. \sin. \alpha^2 = 9,8498673$$

$$C. \log. \cos. d = 0,0954742$$

$$C. \log. \sin. d = 0,2244250$$

$$\log. -0,8003505 = 9,9032802_n$$

$$\log. \sin. f = 9,9573174$$

$$\log. \cos. f = 9,6257563$$

$$C. \log. \cos. f = 0,2991773$$

$$C. \log. \cos. d = 0,0954742$$

$$\log. 0,9500034 = 9,9777252$$

$$\log. \tan. \alpha = 0,1920412$$

$$C. \log. \tan. \alpha = 9,7335137_n$$

$$\log. \sin. f = 9,9369095$$

$$\log. -0,7285585 = 9,8624644_n$$

$$\log. 0,8647876 = 9,9369095$$

$$0,8003505$$

$$0,9500034$$

$$-0,7285585 = 9,8624644_n$$

$$-0,8647876$$

$$\log. 0,1570078 = 9,1969212$$

$$C. \log. 0,1570078 = 0,8030788$$

$$\log. d\psi = 0,2873964$$

$$\log. d\omega = 1,0904752$$

$$d\omega = 12'',3162$$

$$\text{suppositum } \omega = 36^\circ 35' 32'',8200$$

$$\text{correctum } \omega = 36^\circ 35' 45'',1362$$

qvo in locum suppositi substituto, eodemque calculo repetito, prodit ω secunda vice correctum, ab illo infra $0'',003$ diversum. Liceat igitur calculo

hypothesis secundæ enarrando superdere et illud quasi sat exactum amplecti.

$$\log. \tan. (\omega + m') = 0,2026603$$

$$C. \log. \cos. g = 0,3549947$$

$$\log. \tan. N = 0,5576550$$

$$N = 74^\circ 31' 18'',63$$

$$n = 41^\circ 27' 36'',93$$

$$N - n' = 33^\circ 3' 41'',70$$

$$= 119021'',70$$

$$\log. \cos. (\omega + m') = 9,72532$$

$$C. \log. \cos. N = 0,57370$$

$$\log. \cos. g = 9,64501$$

$$\log. \frac{1}{2} = 9,69897$$

$$\log. \tan. q = 9,64300$$

$$C. \log. \cos. q = 0,03836$$

$$\log. \cos. N = 9,42630$$

$$C. \log. \cos. (\omega + m') = 0,27468$$

$$\log. e : \sqrt{2} = 8,75397$$

$$\log. \tan. r = 8,49331$$

$$C. \log. \cos. r^2 = 0,0004210$$

$$\log. (N - n') = 5,0756262$$

$$\log. \cos. g = 9,6450053$$

$$\log. \frac{e^2}{2} : \sqrt{1-e^2} = 7,5082880$$

$$\log. 169'',5667 = 2,2293405$$

$$2'49'',5667$$

$$\log. \cos. g = 9,64501$$

$$\log. \sin. g^2 = 9,90580$$

$$\log. \sin. (N - n') = 9,73683$$

$$\log. \cos. (N + n') = 9,64156$$

$$\log. 0'',0454 = 8,65744_n$$

$$\log. \cos. \ell^2 = 9,9995774$$

$$\log. \cos. h^2 = 0,1506156$$

$$\log. \sin. g^2 = 9,9058016$$

$$\log. \frac{e^2}{4} : (1 - ee) = 7,2097136$$

$$\log. 219'',6279 = 2,3416875$$

$$3'39'',6279$$

$$\log. \sin. n^3 = 9,46277$$

$$\log. \cos. n^4 = 9,87472$$

$$C. \log. \cos. n^2 = 1,14887$$

$$C. \log. \sin. n^2 = 0,03198$$

$$\log. \tan. x = 0,51834$$

$$x = 73^\circ 8',11$$

$$n = 74\ 32',92$$

$$n-x = 1^\circ 24',81$$

$$\log. \sin. (n-x) = 8,39214$$

$$\log. \sin. n^2 = 9,96803$$

$$\log. \cos. n = 9,42557$$

$$C. \log. \cos. x = 0,53743$$

$$\log. \sin. g^4 = 9,81160$$

$$\log. \frac{e^4}{32} : \sin. 1'' = 9,42721$$

$$\log. 0'',0036 = 7,56198$$

$$33^\circ 5'18'',5235$$

$$+ 3 39 ,6279$$

$$+ 0 ,0036$$

$$\sigma = 33^\circ 8'58'',1550$$

$$= 119338'',1550$$

$$\log. \sigma = 5,076779319$$

$$\log. b = 6,513351410$$

$$\log. \sin. 1'' = 4,685574867$$

$$\log. \Sigma = 6,275705596$$

$$\Sigma = 1886711,931$$

valor ipsius Σ exacto ω superstructus.

$$\log. \cos. g = 9,645005303$$

$$C. \log. \cos. f = 0,299189485$$

$$\log. \sin. \alpha = 9,944194788$$

$$\alpha = 118^\circ 25' 40'', 4418$$

valor ipsius α ab exacto ω deductus.

Igitur signis restitutis habentur quantitates quæsitæ

$$\omega' = 36^\circ 35' 45'', 1362$$

$$\phi'' = 59^\circ 56' 23'', 0488$$

$$\Sigma' = 1886711,931$$

$$\alpha' = 118^\circ 25' 40'', 4418$$

Negotium secundum.

Translatis $\phi'' \alpha'' \Sigma'' \phi''' \alpha''' \omega''$
in $\phi' \alpha' \Sigma \phi \alpha \omega$

$$\log. \Sigma = 6,463506871$$

$$C. \log. b = 3,486648591$$

$$C. \log. \sin. i'' = 5,314425133$$

$$\log. \sigma = 5,264580595$$

$$\sigma = 183899'', 5195$$

$$\alpha = 51^\circ 4' 59'', 5195$$

$$\log. \sin. \alpha' = 9,916380867$$

$$\log. \cos. f' = 9,700810515$$

$$\log. \cos. g = 9,617191382$$

$$\log. \tan. f' = 0,236103130$$

$$C. \log. \cos. \alpha' = 0,247693877$$

$$\log. \tan. n' = 0,483797007$$

$$n' = 108^\circ 10' 20'', 4745$$

$$\log. \sin. \sigma = 9,8910125$$

$$log. \cos. (\sigma + 2n') = 8,6520180_n$$

$$C. \log. \sigma = 4,7354194$$

$$C. \log. \sin. i'' = 5,3144251$$

$$\log. \tan. c^2 = 8,5928750$$

$$\log. \tan. c = 9,2964375$$

$$C. \log. \cos. c^2 = 0,0166836$$

$$\log. \sqrt{8:3} = 0,21298$$

$$\log. \sin. \sigma + n' = 9,54925$$

$$\log. \tan. \alpha = 9,76223$$

$$C. \log. \cos. \alpha = 0,06267$$

$$\log. e\sqrt{3:4} = 8,54098$$

$$\log. \sin. g = 9,95913$$

$$\log. \sin. b = 8,56279$$

$$\log. \cos. b^2 = 9,9994196$$

$$\log. \sin. g^2 = 9,9182679$$

$$\log. \frac{e^2}{4} : (1 - ee) = 7,2097136$$

$$C. \log. \cos. c^2 = 0,0166836$$

$$\log. \sigma = 5,2645806$$

$$\log. 256'', 2508 = 2,4086653$$

$$4'16'', 2508$$

$$\log. \sin. n'^3 = 9,93334$$

$$\log. \cos. n' = 9,49398_n$$

$$C. \log. \cos. (\sigma + n')^2 = 0,05822$$

$$C. \log. \sin. (\sigma + n')^2 = 0,90150$$

$$\log. \tan. X = 0,38704_n$$

$$X = 112^\circ 18', 11$$

$$\sigma + n' = 159^\circ 15', 33$$

$$\sigma + n' - X = 46^\circ 57', 22$$

$$\log. \sin. (\sigma + n' - X) = 9,86380$$

$$\log. \sin. (\sigma + n')^2 = 9,09850$$

$$\log. \cos. (\sigma + n') = 9,97089_n$$

$$C. \log. \cos. X = 0,42081_n$$

$$\log. \sin. g^4 = 9,83653$$

$$\log. \frac{e^4}{32} : \sin. i'' = 9,42721$$

$$\log. 0'', 0415 = 8,61774$$

$$\begin{aligned}
 \log. e^{\frac{e^2}{z}} : \sin. r'' &= 51^\circ 4'59'',5195 \\
 - &\quad 416,2508 \\
 - &\quad 0,0415 \\
 l &= 51^\circ 0'43'',2272 \\
 n' &= 108 10 20 ,4745 \\
 n &= 159^\circ 11' 3'',7017 \\
 \log. \sin. n &= 9,550671111 \\
 \log. \sin. g &= 9,959133975 \\
 \log. \sin. f &= 9,509805086 \\
 \log. \tang. f &= 9,533801126 \\
 C.\log. \sqrt{1-e^2} &= 0,001403214 \\
 \log. \tang. \varphi &= 9,535204340 \\
 \varphi &= 18^\circ 55'42'',0006 \\
 \log. \tang. n &= 9,579985524 \\
 \log. \cos. g &= 9,617191382 \\
 \log. \tang. m &= 9,197176906 \\
 m &= 171^\circ 3'5'',5542 \\
 \log. \tang. \alpha' &= 0,164074744 \\
 \log. \sin. f' &= 9,936913645 \\
 \log. \tang. m' &= 0,100988389 \\
 m' &= 128^\circ 23'51'',4213 \\
 d &= m - m' = 42 39 14, 1329 \\
 \log. e : \sqrt{8} &= 8,45294 \\
 \log. \cos. g &= 9,61719 \\
 \log. \tang. f &= 8,07013 \\
 C.\log. \cos. f^2 &= 0,0000600 \\
 \log. l &= 5,2639749 \\
 \log. \cos. g &= 9,6171914 \\
 \log. \frac{e^2}{z} : \sqrt{1-e^2} &= 7,5082880 \\
 \log. 245,1965 &= 2,3895143 \\
 - 4'5'',1965 &
 \end{aligned}$$

$$\begin{aligned}
 \log. \frac{e^2}{z} : \sin. r'' &= 9,72824 \\
 \log. \cos. g &= 9,61719 \\
 \log. \sin. g^2 &= 9,91827 \\
 \log. \sin. l &= 9,89058 \\
 \log. \cos. (n+n') &= 8,66387_n \\
 \log. 0,0066 &= 7,81815_n \\
 \log. 185,0 &= 9,1-0,00001 \\
 d &= 42^\circ 39'14'',1329 \\
 185,0 &= 4,5,1965 \\
 + &\quad 0,0066 \\
 185,0 &= 4,5,1965 \\
 \omega &= 42^\circ 35'8'',9430 \\
 \log. \cos. g &= 9,617191382 \\
 C. \log. \cos. f &= 0,023996040 \\
 \log. \sin. \alpha &= 9,641187422 \\
 \alpha &= 25^\circ 57'28'',5206 \\
 \text{signis restitutis habentur quantitates quæsitæ:} \\
 \varphi''' &= 18^\circ 55'42'',0006 \\
 \omega''' &= 42 35 8 ,9430 \\
 \alpha''' &= 25 57 28 ,5206
 \end{aligned}$$

Negotium tertium.

$$\begin{aligned}
 \omega''' &= 36^\circ 35'45'',1362 \\
 \omega'' &= 42 35 8 ,9430 \\
 \omega''' &= 79^\circ 10'54'',0792 \\
 \text{Translati signis:} \\
 \varphi' \quad \varphi''' \quad \omega''' \quad \alpha''' \quad \alpha''' \quad \Sigma''' \quad \text{in} \\
 \varphi \quad \varphi' \quad \omega \quad \alpha' \quad \alpha \quad \Sigma
 \end{aligned}$$

$$\log. \tan. f = 9,86834$$

$$C. \log. \cos. \omega = 0,72654$$

$$\log. \tan. k = 0,59488$$

$$k = 75^\circ 44',36$$

$$f' = 18^\circ 52',30$$

$$k - f' = 56^\circ 52',06$$

$$\log. \sin. (k - f') = 9,92294$$

$$C. \log. \cot. \omega = 0,28124$$

$$C. \log. \cos. k = 0,60848$$

$$\log. \cot. \alpha' = 9,81266$$

Unde suppositus ipsius α' valor.

$$\log. \sin. \alpha' = 9,92355$$

$$\log. \cos. f' = 9,97600$$

$$\log. \cos. g = 9,89955$$

$$\log. \tan. f' = 9,53380$$

$$C. \log. \cos. \alpha' = 0,26397$$

$$\log. \tan. n' = 9,79759$$

$$n' = 32^\circ 6',41$$

$$\log. \tan. \alpha' = 0,18734$$

$$\log. \sin. f' = 9,50980$$

$$\log. \tan. m' = 9,69714$$

$$m' = 26^\circ 28',14$$

Unde $\alpha' + m' = 32^\circ 6',41$

$$\log. \tan. (\omega + m') = 0,55260_n$$

$$C. \log. \cos. g = 0,10045$$

$$\log. \tan. N = 0,65305_n$$

$$N = 102^\circ 32',00$$

$$n' = 32^\circ 6',41$$

$$N - n' = 70^\circ 25',59$$

$$= 253535'',4$$

$$\log. \cos. (\omega + m') = 9,431_n$$

$$C. \log. \cos. N = 0,664_n$$

$$\log. \cos. g = 9,900$$

$$\log. \frac{g}{\sqrt{2}} = 9,699$$

$$\log. \tan. q = 9,694$$

$$C. \log. \cos. q = 0,047$$

$$\log. \cos. N = 9,336_n$$

$$C. \log. \cos. (\omega + m') = 0,569_n$$

$$\log. e : \sqrt{2} = 8,754$$

$$\log. \tan. r = 8,706$$

$$C. \log. \cos. r^2 = 0,00112$$

$$\log. (N - n') = 5,40404$$

$$\log. \cos. g = 9,89955$$

$$\log. \frac{e^2}{2} : \sqrt{1-e^2} = 7,50829$$

$$\log. 650'',13 = 2,81300$$

$$= 3410''50'',13$$

$$= 0,8888000,0$$

$$\log. \frac{e^4}{16} : \sin. 1'' = 9,728$$

$$C. \log. \cos. g = 9,900$$

$$\log. \sin. g^2 = 9,569$$

$$\log. \sin. (N - n') = 9,974$$

$$\log. \cos. (N + n') = 9,847_n$$

$$\log. 0'',10 = 9,018_n$$

$$= 0,8888000,0$$

$$\log. 410'',13 = 2,81300$$

$$= 3410''50'',13$$

$$= 0,8888000,0$$

$$d = 79^\circ 21'44'',11$$

$$\log. \tan. f = 9,8683384$$

$$C. \log. \cos. d = 0,7337709$$

$$\log. \tan. k = 0,6021093$$

$$k = 75^{\circ} 57' 55'', 03$$

$$f' = 18^{\circ} 52' 17'', 77$$

$$k-f' = 57^{\circ} 5' 37'', 26$$

$$\log. \sin. (k-f') = 9,9240517$$

$$\log. \cot. d = 9,2737577$$

$$C. \log. \cos. k = 0,6152709$$

$$\log. \cot. \alpha' = 9,8130803$$

hinc valor ipsius α' prima vice correctus.

$$\log. \sin. \alpha' = 9,9234243$$

$$\log. \cos. f' = 9,9760040$$

$$\log. \cos. g = 9,8994283$$

$$\log. \tan. f' = 9,5338011$$

$$C. \log. \cos. \alpha' = 0,2634954$$

$$\log. \tan. n' = 9,7972965$$

$$n' = 32^{\circ} 5' 21'', 96$$

$$\log. \tan. \alpha' = 0,1869197$$

$$\log. \sin. f' = 9,5098051$$

$$\log. \tan. m' = 9,6967248$$

$$m' = 26^{\circ} 26' 48'', 31$$

$$\log. \tan. (\omega+m') = 0,5532451$$

$$C. \log. \cos. g = 0,1005717$$

$$\log. \tan. N = 0,6538168$$

$$N = 102^{\circ} 30' 42'', 97$$

$$n' = 32^{\circ} 5' 21'', 96$$

$$N-n' = 70^{\circ} 25' 21'', 01$$

$$= 253521'', 01$$

$$\log. \cos. (\omega+m') = 9,43039$$

$$C. \log. \cos. N = 0,66426$$

$$\log. \cos. g = 9,89943$$

$$\log. \tan. \frac{1}{2} = 9,69897$$

$$\log. \tan. q = 9,69305$$

$$C. \log. \cos. q = 0,04728$$

$$\log. \cos. N = 9,33574$$

$$C. \log. \cos. (\omega+m') = 0,56961$$

$$\log. e : \sqrt{2} = 8,75397$$

$$\log. \tan. r = 8,70660$$

$$C. \log. \cos. r^2 = 0,0011232$$

$$\log. N-n' = 5,4040139$$

$$\log. \cos. g = 9,8994283$$

$$\log. \frac{e^2}{2} : \sqrt{1-e^2} = 7,5082880$$

$$\log. 649'', 9103 = 2,8128534$$

$$649'', 9103 = 2,8128534$$

$$\log. \frac{e^4}{16} : \sin. 1'' = 9,72824$$

$$\log. \cos. g = 9,89943$$

$$\log. \sin. g^2 = 9,56902$$

$$\log. \sin. (N-n') = 9,97414$$

$$\log. \cos. (N+m') = 9,84644$$

$$\log. o'', 1041 = 9,01727$$

$$\omega = 79^{\circ} 10' 54'', 0792$$

$$+ 1049,9103$$

$$o, 1041$$

$$d = 79^{\circ} 21' 43'', 8854$$

$$\log. \tan. f = 9,868338382$$

$$C. \log. \cos. d = 0,73376814$$

$$\log. \tan. k = 0,602106796$$

$$k = 75^{\circ} 57' 54'', 7539$$

$$f' = 18^{\circ} 52' 17'', 7665$$

$$k-f' = 57^{\circ} 5' 36'', 9874$$

$$\log. \sin. (k-f') = 9,924051355$$

log. cos. $g = 9,899428245$	alium lēp. $\alpha''' = 80^\circ 27' 7'',9800$
C. log. cos. $f = 0,094513683$	$\alpha' = 33^\circ 17' 37'',7248$
log. sin. $\alpha = 9,993941928$	$\beta''' = 47^\circ 9' 30'',2552$
	$\alpha = 80^\circ 27' 7'',9800$
Signis restitutis habentur quæsitæ	$360^\circ = 359^\circ 59' 60'',0000$
$\alpha''' = 56^\circ 57' 57'',8587$	$\alpha' = 118^\circ 25' 40'',4418$
$\Sigma''' = 4020210,440$	$\alpha'' = 124^\circ 25' 32'',0883$
$\alpha''' = 80^\circ 27' 7'',9800$	$\beta''' = 117^\circ 8' 47'',4699$

Cifras posteriores duas, etsi, illæsa veritate ad praxin necessaria, haud dubie ubivis negligi posint, in hocce tamen exemplo, ut calculi, qvo valores art. 16 revera eruti sunt, indicetur species, aservari placuit.

Negotium quartum.

$$\begin{aligned} m\alpha''' &= 56^\circ 57' 57'', 8587 \\ m\alpha'' &= 25 \quad 57 \quad 28, 5206 \\ \beta' &= 31^\circ 8' 29'', 3381 \end{aligned}$$

19.

Casus in art. 17 expositi omnium simplicissimi videntur ideoqve casus primi ordinis appellandi. Casus secundi ordinis dicentur ii, qvi omnia præter unum data elementa cum illis communia habent, et alter primi ordinis alteri secundi ordinis respondens dicetur, ubi inter utrumqve hæc exstat necessitudo, ut respectu qvatuor elementorum convenient, respectu unius differant. Qvivis igitur propositus secundi ordinis casus solutionem juxta præcepta art. 17 conseqvi poterit, si modo ad respondentem reductus fuerit; sed erit reductus, ubi elementum respondenti proprium inventum est. Qvarè peculiari respondentis elemento arbitrarium, qvineciam approximatum, qvalis fere semper nobis in promptu erit, valorem assignantes, peculiare casus propositi elementum huic hypothesi convenienter calculo eruanus, simulqve observemus, qvanta sit differentia inter datum et computatum hujus valorem. Deinde alterum peculiare ratione alterius differentietur, et correctione peculiari respondentis elemento admovenda denotetur. Qvo eodem cursu saepius, si opus fuerit repetito, verum tandem elementum respondenti proprium conseqvemur.

Casus tertii ordinis dicentur, qvi omnia, duobus exceptis, data elementa cum illis primi ordinis communia habent. Similiter casus primi ordinis casui tertii ordinis respondens dicetur, si qvoad tria elementa consentiunt, qvoad duo dissentiunt. Qvicunqve igitur tertii ordinis casus, ad respondentem reductus, solutionem ex præceptis art. 17 sortietur. Proinde pro utroqve respondentis peculiari elemento factis, modo veritatem haud nimis excedentibus, valoribus adoptatis, jam licebit, via qvam præmonstrat art. 17, utrumqve casus propositi peculiare elementum investigare. Tum valores ita erutos cum valoribus eorundem elementorum datis conferre conveniet, nec non utrumqve respondentis peculiare elementum ratione utriusqve proposito casui peculiaris differentiare. Exinde binæ emergent æquationes, in qvibus duo ignota differentialia elementorum respondentis peculiariū ita cum quantitatibus notis mista occurrunt, ut, altero ignoto differentiali eliminato, alterum nobis sese offerat. Utroqve comperto et ad peculiaria respondentis elementa emendanda adhibito, correctiora hæcce prodibunt, atqve cursu eodem repetito, tandem adeo correcta, ut reductio absoluta censi possit.

20.

Ut art. præc. similiter exemplo illustretur, jam calculus, qvo casus tertii ordinis ad respondentem primi ordinis reducitur, summatim est exhibendus.

Litteris græcis ac latinis in eadem significatione ac supra adhibitis, sint datae
 $\beta' = 31^\circ 0' 29'', 34$, $\beta'' = 47^\circ 9' 30'', 26$, $\beta''' = 117^\circ 8' 47'', 47$, $\phi'' = 66^\circ 55' 43'', 41$, $\phi''' = 994509$ tois.
 et quæsita respondentis elementa

$\phi' = 50^\circ 55' 55'', 48$, $\alpha' = 33^\circ 23' 53'', 28$

et conjiciendo supponetur

$$\phi'' = 54^\circ 3' 46''$$

imediatum est ut secundum hanc hypothesem illud exinde nullum nec non denotabitur

$$d\phi'' = - \frac{\cos \phi''}{\sin \alpha''} d\alpha'' \left\{ \frac{\cos \phi''' \tan \alpha''}{\sin \alpha'' \cos \phi''} - \cos \alpha'' \tan \psi'' \tan \omega'' + \sin \phi''' \right\}$$

qua formula differentiali conjectura emendari poterit.

Principia momenta quatuor hypothesum, quae omnes ad sphæram spectant, in conspectu seqventi exhibentur.

	I.	II.	III.	IV.
ϕ''	$54^\circ 3' 46''$	$63^\circ 19' 47''$	$60^\circ 48' 10''$	$60^\circ 6' 13'', 23$
α''	$125^\circ 20' 10''$	$112^\circ 16' 4''$	$116^\circ 47' 22''$	$117^\circ 52' 41'', 09$
α'''	$117^\circ 31' 30''$	$130^\circ 35' 8''$	$126^\circ 3' 50''$	$124^\circ 58' 31'', 44$
ω''	$54^\circ 0' 44''$	$32^\circ 0' 22''$	$40^\circ 20' 21''$	$42^\circ 11' 46'', 47$
ω'''	$65^\circ 17' 48''$	$35^\circ 18' 38''$	$48^\circ 4' 55''$	$50^\circ 46' 37'', 30$
$d\alpha''$	$- 14^\circ 21' 52''$	$15^\circ 37' 18''$	$2^\circ 51' 1''$	$9^\circ 18' 18''$
$d\phi''$	$9^\circ 15' 52''$	$- 2^\circ 31' 37''$	$- 41' 57''$	$- 2^\circ 33' 24''$

itaque ille ex trig. sphæric. deductus valor ipsius $\phi'' = 60^\circ 3' 39'', 99$

et per directam solutionem triang. sphæric. approximate habetur $\phi' = 36^\circ 46' 40'', 10$

Qvo quasi præcursorio calculo absolute, introducentes deinde

$$a = \sin_{\alpha''} (\cos l'' + \cot_{\alpha''} \tan_{\alpha''} \alpha'') \quad b = \frac{\sin l''}{\cot_{\alpha''}} \left\{ \tan f'' - \frac{\cot_{\alpha'} \cos(\alpha' - \alpha'')}{\sin l' \sin_{\alpha'}} \right\}$$

$$c = \sin_{\alpha''} \cot l'' - \tan f'' \tan_{\alpha''} \alpha'' \quad d = \sin_{\alpha''} \cot l'' - \sin_{\alpha''} \cot l''$$

$$e = \sin_{\alpha''} \cot l''$$

$$f = \frac{\sin_{\alpha'}}{\sin l'} \left\{ \cos l' - \frac{\sin l'' \sin_{\alpha''}}{\sin_{\alpha''} \sin l''} \right\}$$

$$g = - \frac{e}{\cos l''}$$

$$h = \frac{\sin_{\alpha'} \sin l''}{\sin l' \sin_{\alpha''}} d$$

$$k = \frac{\sin_{\alpha''} (a + b)}{\sin_{\alpha''} \sin l''}$$

$$l = \frac{\cos_{\alpha''} \sin_{\alpha''} \cos(\alpha' + \alpha'')}{\cos f'' \sin d'' \cos_{\alpha''} \sin_{\alpha'}}$$

$$m = \frac{\cos(\alpha' - \alpha'')}{\sin l' \sin_{\alpha'}}$$

$$n = \frac{ac + bd}{\sin_{\alpha''}}$$

$$r = \tan f'' \tan_{\alpha''} \alpha''$$

$$s = \frac{\tan_{\alpha''}}{\tan_{\alpha''}} \cdot l$$

eliciemus

$$df'' = \frac{d\beta' (e - f) - d\beta'' (g - h)}{(n + r - s)(e - f) + (f + l - m)(g - h)}$$

$$df' = \frac{d\beta' (f + l - m) + d\beta'' (n + r - s)}{(n + r - s)(e - f) + (f + l - m)(g - h)}$$

qvæ formulæ differentiales valores jamjam approximatos correctiores reddent. Ipso
suis autem calculi per tres hypotheses continuati præcipua momenta ita se habent:

	I.	II.	III.
f''	$59^{\circ} 58' 51'', 54$	$59^{\circ} 57' 58'', 24$	$59^{\circ} 51' 34'', 22$
f'	$36^{\circ} 41' 20'', 66$	$36^{\circ} 39' 32'', 90$	$36^{\circ} 26' 40'', 32$
ω'	$65^{\circ} 19' 54'', 52$	$65^{\circ} 18' 48'', 00$	$65^{\circ} 10' 49'', 30$
α'	$36^{\circ} 45' 7'', 57$	$36^{\circ} 43' 58'', 95$	$36^{\circ} 35' 45'', 32$
α''	$33^{\circ} 22' 17'', 65$	$33^{\circ} 21' 42'', 89$	$33^{\circ} 17' 35'', 52$
α'''	$118^{\circ} 4' 44'', 67$	$118^{\circ} 11' 17'', 18$	$118^{\circ} 25' 42'', 78$
ω''	$42^{\circ} 16' 46'', 98$	$42^{\circ} 19' 15'', 98$	$42^{\circ} 35' 8'', 11$
ω'''	$124^{\circ} 46' 27'', 86$	$124^{\circ} 39' 55'', 35$	$124^{\circ} 25' 29'', 75$
f'''	$18^{\circ} 59' 38'', 94$	$19^{\circ} 5' 50'', 31$	$18^{\circ} 52' 24'', 04$
α'''	$25^{\circ} 45' 35'', 75$	$25^{\circ} 49' 33'', 70$	$25^{\circ} 57' 30'', 07$
ω'''	$79^{\circ} 1' 54'', 55$	$79^{\circ} 3' 0'', 93$	$79^{\circ} 10' 53'', 43$
α'''	$56^{\circ} 46' 6'', 46$	$56^{\circ} 49' 41'', 07$	$56^{\circ} 58' 1'', 96$
α'''	$89^{\circ} 30' 41'', 50$	$80^{\circ} 23' 57'', 25$	$80^{\circ} 27' 2'', 04$
$d\beta''$	$1^{\circ} 6', 41$	$7^{\circ} 15', 90$	$3^{\circ}, 74$
$d\beta'$	$- 1^{\circ}, 37$	$21^{\circ}, 97$	$- 2^{\circ}, 55$
df''	$- 53', 30$	$6' 23', 92$	$- 0', 65$
df'	$- 1' 47', 76$	$12' 52', 58$	$+ 1', 46$

Igitur desiderata respondentis elementa

$$f'' = 59^{\circ} 51' 33'', 57 \quad f' = 36^{\circ} 26' 41'', 78$$

"φ'	φ"	ω'	3	φ"	ω'	"Σ'	45	φ'	φ'	φ"	φ"	φ'''	φ'''	2		
φ'	φ"	ω'	15	29	φ"	ω'	Σ'	12	8	φ'	,φ'	ω'	,φ'''	ω'''	3	
φ'	φ"	Σ'	41	32	φ"	ω'	ψ'	28	58	φ'	φ'	ω'	φ'''	ω'''	29	
"φ'	φ"	"Σ'	4	φ"	ω'	Σ'	23	22	φ'	φ'	Σ'	φ"	Σ"	32		
,φ'	φ"	,Σ'	9	φ"	ω'	,Σ'	18	51	φ'	,φ'	,Σ'	,φ'''	,Σ'''	4		
φ'	φ"	ψ'	24	35	φ"	ω'	,Σ'	53	30	φ'	,φ'	,Σ'	,φ'''	,Σ'''	35	
φ'	ω'	ω'	16	φ"	ω'	ψ'	62	59	φ'	φ'	ψ'	φ"	ψ''	φ'''	ψ'''	1
φ'	ω'	Σ'	40	φ"	Σ'	,Σ'	43	52	φ'	φ"	ω'	φ'''	ω'''	φ'''	ω'''	19
φ'	ω'	Σ'	10	φ"	Σ'	,Σ'	54	33	φ'	φ"	ω'	φ'''	ω'''	φ'''	ω'''	19
φ'	ω'	ψ'	27	φ"	Σ'	ψ'	64	60	φ'	φ"	Σ'	φ'''	Σ"	φ'''	Σ'''	21
φ'	ω'	Σ'	50	φ"	Σ'	,Σ'	13	7	φ'	φ"	Σ'	φ'''	Σ"	φ'''	Σ'''	5
φ'	ω'	Σ'	17	φ"	Σ'	ψ'	25	61	φ'	φ"	,Σ'	φ'''	,Σ''	φ'''	,Σ'''	6
φ'	ω'	,Σ'	31	φ"	Σ'	ψ'	68	36	φ'	φ"	ψ'	φ'''	ψ''	φ'''	ψ'''	57
φ'	ω'	ψ'	63	ω'	ω'	Σ'	49	φ'	ω'	ω'	ω''	ω'''	ω'''	ω'''	20	
φ'	Σ'	,Σ'	42	ω'	ω'	,Σ'	47	φ'	ω'	Σ'	ω''	Σ''	ω'''	Σ'''	38	
φ'	Σ'	,Σ'	34	ω'	ω'	,Σ'	55	φ'	ω'	,Σ'	ω''	,Σ''	ω'''	,Σ'''	45	
φ'	Σ'	ψ'	65	ω'	ω'	ψ'	66	φ'	ω'	,Σ'	ω''	,Σ''	ω'''	,Σ'''	8	
φ'	Σ'	,Σ'	11	ω'	Σ'	,Σ'	44	φ'	ω'	ψ'	ω''	ψ''	ω'''	ψ'''	58	
φ'	,Σ'	ψ'	26	ω'	Σ'	,Σ'	56	φ'	ω'	Σ'	ω''	Σ''	ω'''	Σ'''	22	
φ'	,Σ'	ψ'	37	ω'	Σ'	ψ'	67	φ'	ω'	,Σ'	ω''	,Σ''	ω'''	,Σ'''	51	
φ"	ω'	ω'	14	20	ω'	Σ'	,Σ'	46	φ'	ω'	,Σ'	ω''	,Σ''	ω'''	,Σ'''	30
φ"	ω'	Σ'	39	38	ω'	Σ'	ψ'	48	φ'	ω'	ψ'	ω''	ψ''	ω'''	ψ'''	59
					ω'	,Σ'	ψ'	69	φ'	Σ'	,Σ'	Σ''	,Σ''	Σ'''	,Σ'''	52
									φ'	Σ'	,Σ'	Σ''	,Σ''	Σ'''	,Σ'''	33
									φ'	Σ'	ψ'	Σ''	ψ''	Σ'''	ψ'''	60
									φ'	Σ'	,Σ'	Σ''	,Σ''	Σ'''	,Σ'''	7
									φ'	Σ'	ψ'	Σ''	ψ''	Σ'''	ψ'''	61
									φ'	Σ'	ψ'	Σ''	ψ''	Σ'''	ψ'''	36
									φ"	β'''		φ'''	β''	φ'''	β''	29
									φ'''	β'''		φ'''	β''	φ'''	β''	19
									ω''	β'''		ω'''	β''	ω'''	β''	20
									Σ''	β'''		Σ'''	β''	Σ'''	β''	22
									Σ''	β'''		Σ'''	β''	Σ'''	β''	51
									Σ''	β'''		Σ'''	β''	Σ'''	β''	30
									ψ''	β'''		ψ'''	β''	ψ'''	β''	59